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Transient Stability-Constrained Optimization for Power System Dispatch and Operational Control

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1 **Stability Preliminaries**

- Definition & Classification
- Challenges
- Assessment & Control
- OPF, SCOPF, TSCOPF (Transient Constrained-OPF)

2 **Literature Review**

- Direct Method
- Discretization Method
- Data-Driven Method
- Evolutionary Algorithm-based Method

3 **Proposed Methods**

- Hybrid method for TSCOPF
- Pattern discovery-based method for TSCOPF
- Decision tree-based method for TSCOPF
- Practical method for TSCUC
- Robust TSCOPF with Uncertain Dynamic Loads
- Robust TSCOPF with Uncertain Wind Power
- Preventive TSC for Wind Power Variation
- Preventive-Corrective Coordinated TSCOPF
- Fully Robust TSCOPF under Wind Uncertainty

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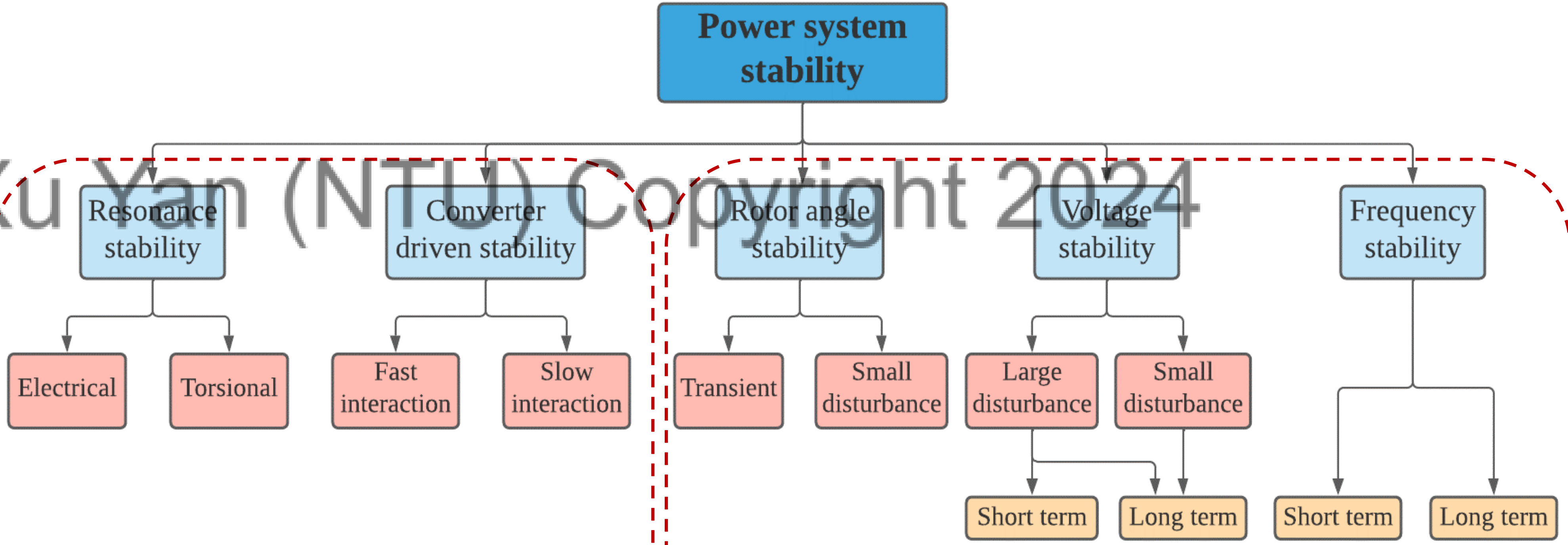
Power System Stability: Definition & Classification

IEEE/CIGRE joint task force:

*"The ability of an electric power system, for a **given initial operating condition**, to **regain a state of operating equilibrium** after being subjected to **a physical disturbance**, with **most system variables bounded** so that practically the entire system remains intact."*



Newly added categories :
N. Hatziargyriou *et al.*, "Definition and Classification of Power System Stability – Revisited & Extended," *IEEE Trans. Power System*, 2021.

Traditional categories:
P. Kundur et al., "Definition and classification of power system stability IEEE/CIGRE joint task force on stability terms and definitions," *IEEE Trans. Power System*, 2004.

1. Stability
- Definition & classification
- **Challenges**
- Assessment & Control
- Optimization Model

2. Review
- Direct Method
- Discretization Method
- Data-Driven Method
- Evolutionary Algorithm

3. Our Methods
- Hybrid Method
- Data-Driven Methods
- TSC-Unit Commitment
- Robust TSCOPF-Load
- Robust TSCOPF-Wind
- PC-CC Cor. TSCOPF
- Full Robust TSCOPF

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Power System Stability: Challenges

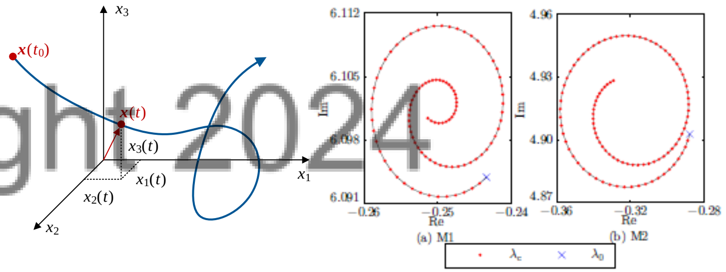
Generation side: Higher-level intermittent renewable energy resources

Demand side: Demand response, distributed energy storage units, etc.

Device-grid interface: Power-electronic converters

Higher operating uncertainties

Complicated system dynamics



- 2003 — North America Blackout
- 2006 — West Europe Blackout
- 2012 — India Blackout
- 2016 — South Australia Blackout
- … ?

Very high wind power penetration level (48%)

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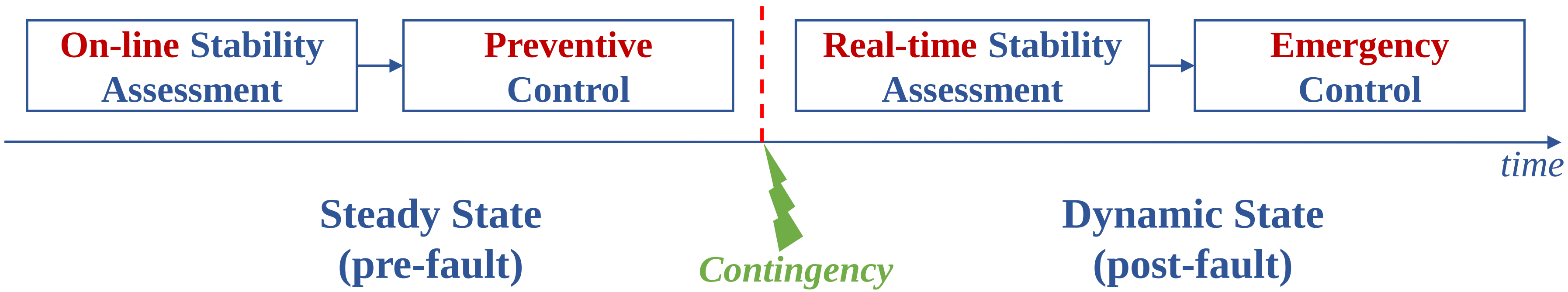
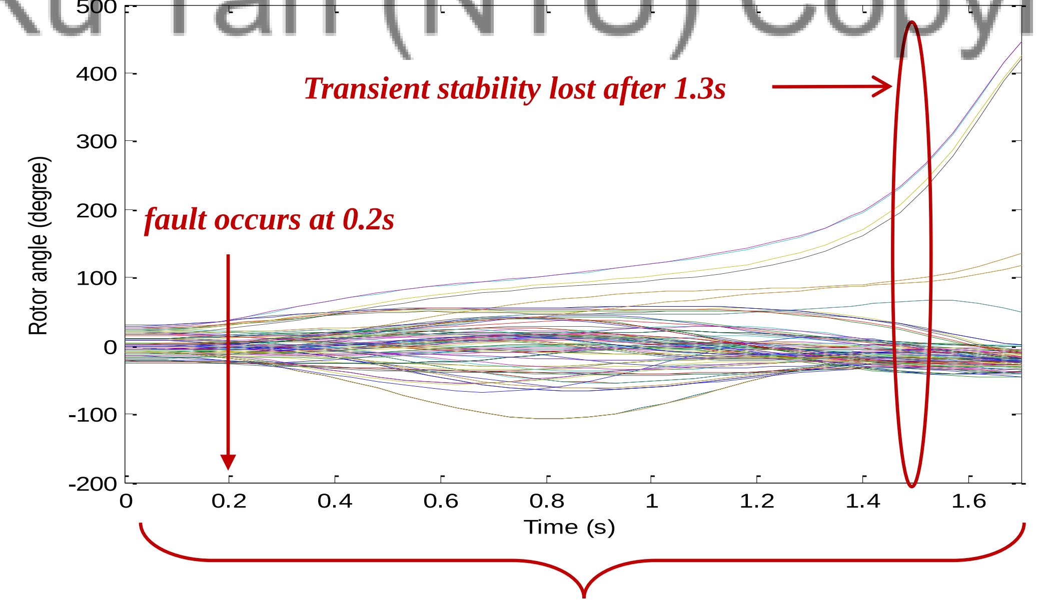
Power System Stability: Assessment & Control



$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{p}, \boldsymbol{\lambda}) \quad 0 = \mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{p}, \boldsymbol{\lambda})$$



PSS/E simulation costs 2.2s CPU time

- **Transient stability**, i.e., the ability of the system to keep synchronism after a large disturbance, is the most stringent requirement for a power system because instability can develop rapidly within several cycles after a disturbance.
- It mainly depends on both its inherent **dynamic characteristics**, i.e., how the system responds to disturbances, and its **steady-state operating conditions**, i.e., how the system is dispatched.
- Its operation control includes **preventive control** (e.g., generation redispatch) and **emergency control** (e.g., load and generation tripping).
- **Wind power generation** adds more complexity due to its stochastic power output nature and power-electronic converter interfacing.

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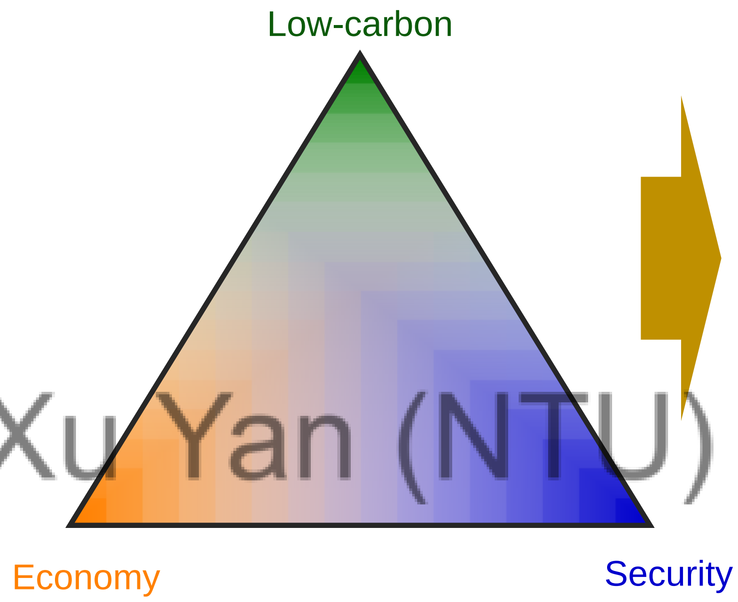
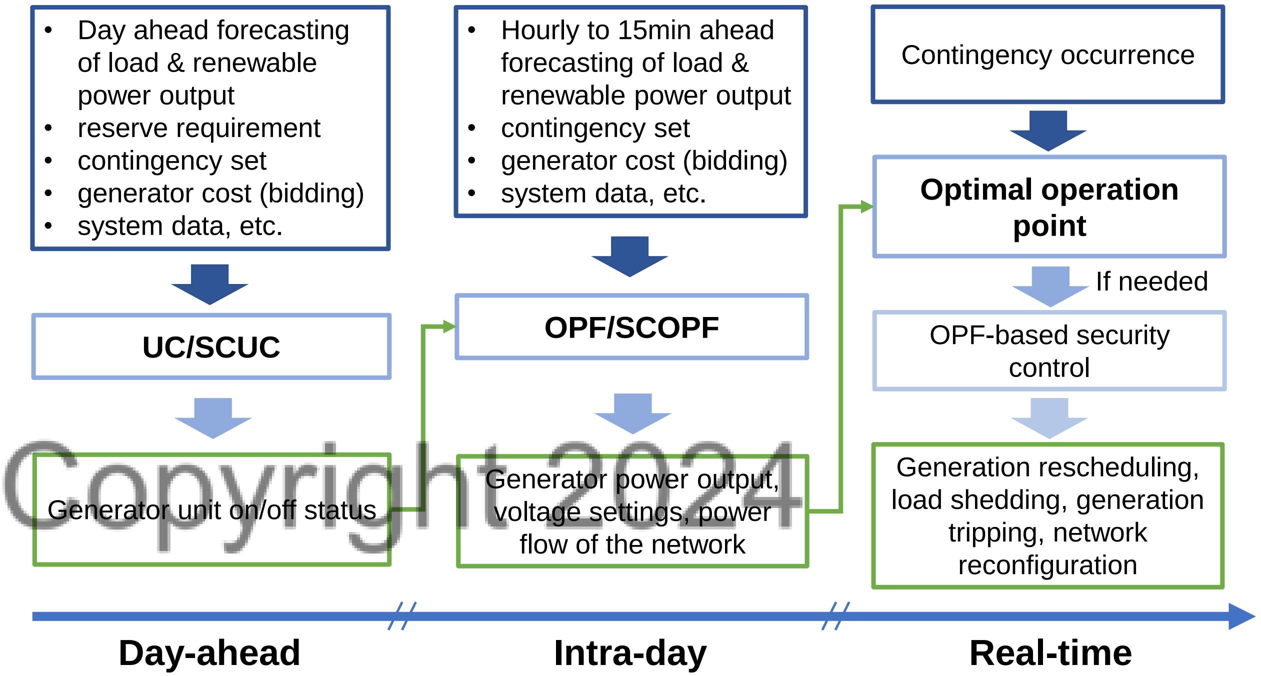
Power System Optimal Operation

Conflicting triangle for power system operation objectives



Power system operation framework



Optimal Power Flow (OPF)

$$\min_{\mathbf{u}} F(\mathbf{x},\mathbf{u},\mathbf{y})$$

$$s.t.\ \ \mathbf{g}(\mathbf{x},\mathbf{u},\mathbf{y})=0$$

$$\mathbf{h}(\mathbf{x},\mathbf{u},\mathbf{y})\le 0$$

- **u – control variables**, such as active power output and voltage settings of the generator units
- **x – state variables** (also called dependent variable), such as bus voltage magnitudes and angles as well as branch power flow;
- **y – parameters**, such as load demand, network topology and network parameters, which can be (or assumed to be) deterministic if they can be accurately predicted such as the load demand; or stochastic if less predictable, such as the wind and solar power output that is naturally uncertain.
- **g – equality constraints**, i.e., power balance based on power flow equations
- **h – inequality constraints**, i.e., network operating limits (such as branch flow limits and voltage limits) and limits on control variables (such as generator capacity).

Optimal Power Flow with Security/Stability Constraints

Security-Constrained OPF (SCOPF)

$$\min_{\mathbf{u}_0} f_0(\mathbf{x}_0, \mathbf{u}_0, \mathbf{y}_0)$$

$$s.t.\ \mathbf{g}_k(\mathbf{x}_k, \mathbf{u}_0, \mathbf{y}_0) = 0 \quad (k = 0, \ldots, K)$$

$$\mathbf{h}_k(\mathbf{x}_k, \mathbf{u}_0, \mathbf{y}_0) \le 0 \quad (k = 0, \ldots, K)$$

- subscript *k* denotes the *k*th system configuration (*k*=0 corresponds to pre-contingency configuration, and *k* >0 corresponds to the *k*th post-contingency configuration).
- SCOPF only considers steady-state security criterion, i.e., branch flow and bus voltage.

Transient Stability-Constrained OPF (TSCOPF)

$$\min F(\mathbf{x}, \mathbf{u}, \mathbf{y})$$

$$s.t.\ \mathbf{g}(\mathbf{x}, \mathbf{u}, \mathbf{y}) = 0$$

$$\mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{y}) \le 0$$

$$\mathbf{TSI}_k(\mathbf{x}, \mathbf{u}) \ge \varepsilon, \{k \in C\}$$

- Constrained by a transient stability index (TSI) larger than a threshold ε for a pre-defined contingency list *C*.
- The TSI corresponds to a large set of differential-algebraic equation (DAE) that characterizes system dynamics.

Challenges for TSCOPF problems

1) **Selection of a proper TSI** → traditional criterion is the maximum rotor angle difference, e.g., 180º, which is however system and operation condition dependent; improper values lead to conservative or optimistic solutions.
2) **Handling of the TSI constraints** → a large set of DAEs that are mathematically intractable in optimization model.
3) **Solution quality** → computation speed, optimality, convergence; compatible with the industry practice (e.g., industry-grade models and simulation tools).
4) **Modeling and addressing the uncertainties** → intermittent renewable energy resources have been well addressed in SCOPF problems, but not TSCOPF. Besides, the load dynamics and its model uncertainty has a significant impact on transient stability. Solutions should be robust against such uncertainties.
5) **Multi-stage coordination for transient stability control** → preventive control (PC) before the contingency and correct control (CC) actions after the contingency.

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The first (probably) paper on TSCOPF – 1983

IEEE Transactions on Power Apparatus and Systems, Vol. PAS-102, No. 7, July 1983 2145

DYNAMIC SECURITY DISPATCH: BASIC FORMULATION

K.S. Chandrashekhar, Member, IEEE D.J. Hill, Member, IEEE

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University of Newcastle
New South Wales, 2308, Australia

Abstract - This paper presents preliminary results on a direct method for dynamic security dispatch in large power systems. The usual indirect approach starts with an operating point chosen to give optimal economy; then contingency testing using simulation indicates adjustments which may be needed to ensure adequate transient stability after the most likely faults. The nature of the adjustments is derived from "intuitive feel" for the system. Using a model given by Bergen and Hill [1], distribution factors are presented for systemmatically improving transient stability with variation of bus powers, line impedances and bus voltages. By incorporating a stability index into the cost function for economic dispatch, there can be a trade-off between the requirements for economy and stability in choosing an operating point. The method could be adapted to either planning or on-line scheduling to ensure adequate dynamic security. The application of the approach is demonstrated on a 5 bus example system.

requirements. Thus, although there has been several discussions of dynamic security in the literature [4, 12-14], there has been no suggestion of a systemmatic algorithm for dynamic security dispatch in the sense required here.

The structure preserving model presented by Bergen and Hill [1] has the significant feature of allowing direct stability analysis on a dynamic model whose steady-state solution is given by a (simplified) load-flow model. Thus it is suitable for the task in hand. Further the results of [1] have provided a convenient algebraic index of stability as a function of injected bus powers, system voltages and line impedances. This can be used directly for stability enhancement or included in an optimal dispatch algorithm cost function. If the techniques are to be used on-line, then we are assuming that a direct stability assessment test has preceded the enhancement to identify the required improvements. A companion paper [15] deals with this topic.

K. S. Chandrashekhar and D. J. Hill, "Dynamic Security Dispatch: Basic Formulation," *IEEE Trans. Power Apparatus and Systems*, 1983.

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Literature Review for TSCOPF

- Direct method (sequential method)
- Discretization method (global method)
- Data-driven (machine learning) method
- Evolutionary algorithm-based method

Y. Xu, Z.Y. Dong, Z. Xu, et al, "Power system transient stability-constrained optimal power flow: a comprehensive review," *Proc. 2012 IEEE PES General Meeting*, San Diego, CA, USA, Jul. 2012.

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Direct method

- To directly stabilize the system through critical control variables (e.g., generation output) based on the underlying stability mechanism.
- The key is to analytically determine the generation shifting amount and direction.

Pros	Cons
• High solution efficiency • Explicit stabilization mechanism	• Local optimal (sub-optimal) solutions • Over stabilization

- Classic references:

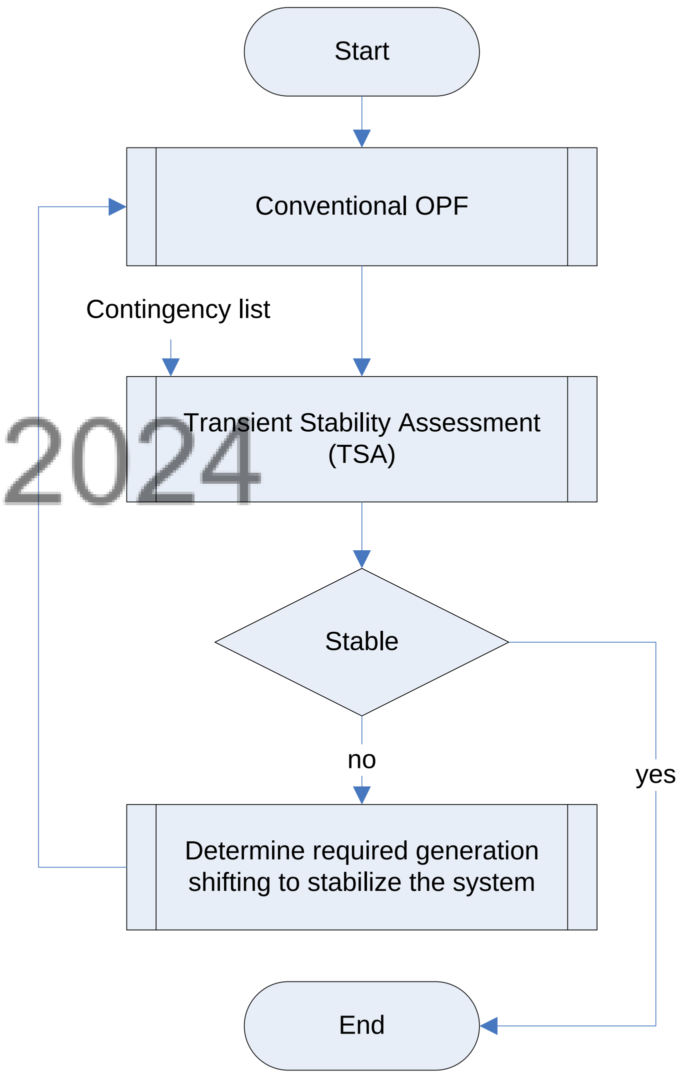
[1] K. S. Chandrashekhar and D. J. Hill, "Dynamic Security Dispatch: Basic Formulation," *IEEE Trans. Power Apparatus and Systems*, 1983.

[2] T. Nguyen and M. A. Pai, “Dynamic security-constrained rescheduling of power systems using trajectory sensitivities,” *IEEE Trans. Power Syst.*, 2003.

[3] D. Ruiz-Vega and M. Pavella, “A comprehensive approach to transient stability control I: Near optimal preventive control,” *IEEE Trans. Power Syst.*, 2003.

[4] R. Zarate-Minano, T. V. Cutsem, F. Milano, and A. J. Conejo, “Securing transient stability using time-domain simulations within an optimal power flow,” *IEEE Trans. Power Syst.*, 2010.

Computation flowchart



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Discretization method

- Model the TSI as the swing equations (DAEs).
- Constrain the rotor angle in Center of Inertia (COI) framework with an empirical threshold.
- Discretize the DAEs into numerically equivalent algebraic equations and incorporate them into the OPF model.

Pros	Cons
• Global solution • Mathematically rigorous	• Heavy computational burden • Convergence difficulty

- Classic references:

[1] D. Gan, R. J. Thomas, and R. D. Zimmerman, "Stability-constrained optimal power flow," *IEEE Trans. Power Syst.*, 2000.

[2] Y. Yuan, J. Kubkawa, and H. Sasaki, "A solution of optimal power flow with multicontingency transient stability constraints," *IEEE Trans. Power Syst.*, 2003.

[3] Q. Jiang and Z. Huang, "An enhanced numerical discretization method for transient stability constrained optimal power flow," *IEEE Trans. Power Syst.*, 2010.

Classic swing equations

$$\begin{cases} M_i \dfrac{d\omega_i}{dt} = P_{mi} - P_{ei} \\ \dfrac{d\delta_i}{dt} = \omega_i \end{cases}, \{i \in S_G\}$$

TSI constraints

$$\delta_i^{COI} = \delta_i^n - \frac{\sum_{i=1}^{S_G} M_i \cdot \delta_i^n}{\sum_{i=1}^{S_G} M_i} < \varepsilon$$

Discretization of DAEs

$$\frac{d\omega_i}{dt} = D_i(P_{mi}, P_{ei})$$

$$\delta_i^{n+1} - \delta_i^n - \frac{h}{2}(\omega_i^{n+1} + \omega_i^n) = 0$$

$$\omega_i^{n+1} - \omega_i^n - \frac{h}{2}(D_i^{n+1} + D_i^n) = 0$$

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Data-driven (machine learning) method

- **Extract stabilization rules via machine learning from a transient stability database and incorporates the rules as explicit constraints into the ordinary OPF model.**
- **The key is to extract effective, accurate, and robust stabilization rules.**

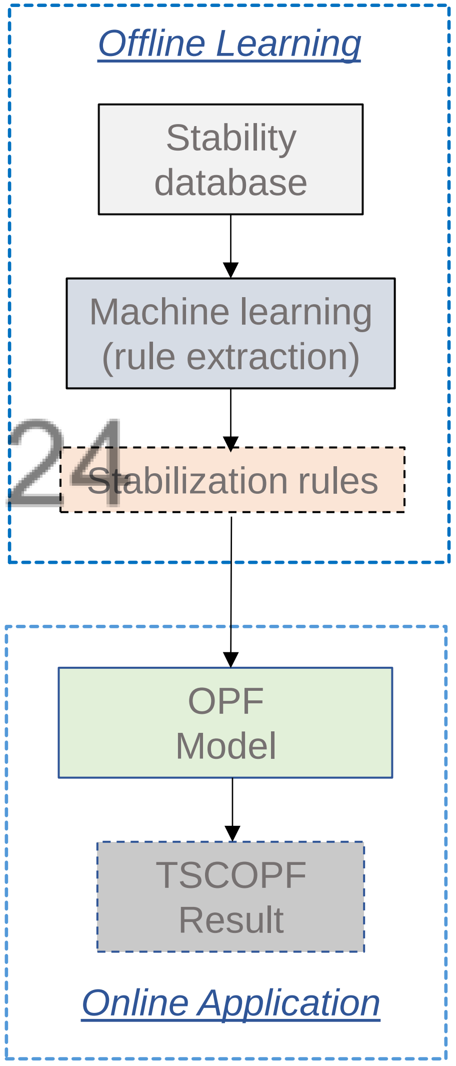
Pros	Cons
• **High online solution speed (as direct methods)** • **The rules can be used for stability assessment/monitoring**	• **The rules depends on database and machine learning** • **Cannot guarantee the effectiveness & accuracy**

- **Classic references:**

[1] E.S. Karapidakis, N.D. Hatziargyriou, "On-Line preventive dynamic security of isolated power systems using decision trees," *IEEE Trans. Power Syst.*, 2002.

[2] I. Genc, R. Diao, V. Vittal, et al, "Decision trees-based preventive and corrective control applications for dynamic security enhancement in power systems," *IEEE Trans. Power Syst.*, 2010.

Computation flowchart



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Evolutionary algorithm (EA)-based method

- Run an EA to heuristically search the optimal solutions of the TSCOPF model, where the stability compliance is checked through TSA.
- The key is to select a powerful EA, proper control variables, and an efficient TSA tool.

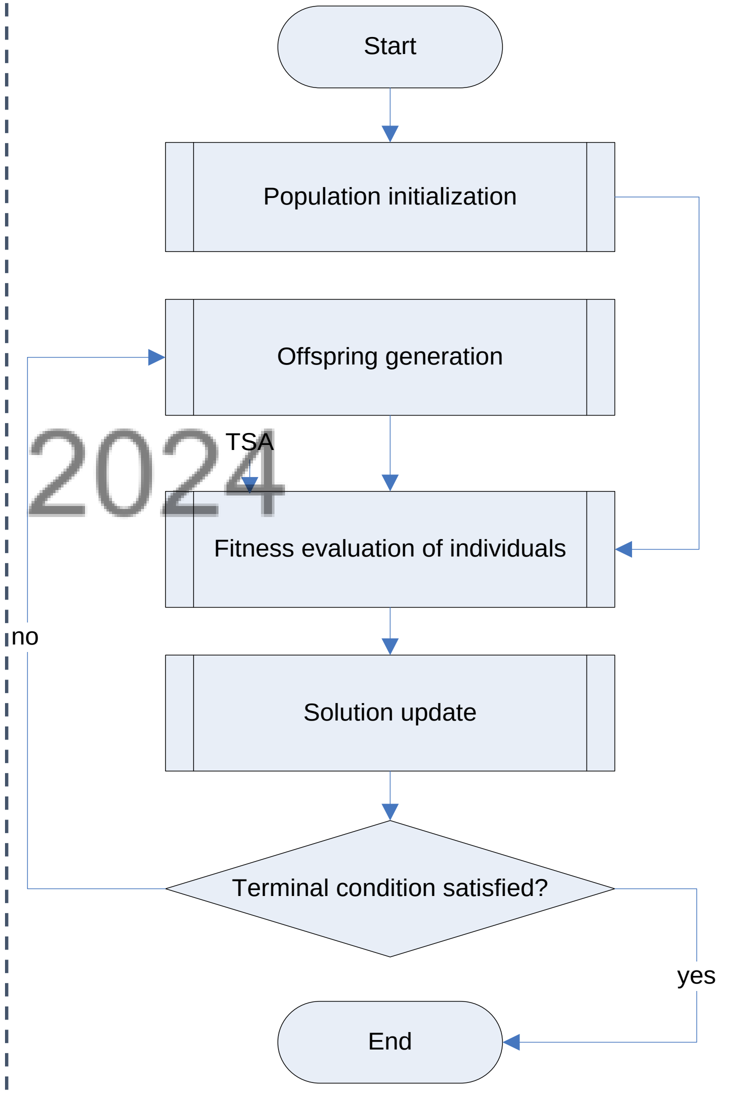
Pros	Cons
• "Global" optimality • No limitation to problem modeling and TSI • Easy to implement • All stability categories can be considered	• Solution inconsistency • Non-rigorous convergence • Long computation time

- Classic references:

[1] N. Mo, Z.Y. Zou, K. W. Chan, and T. Y. G. Pong, "Transient stability constrained optimal power flow using particle swarm optimisation," *IET Gen., Tran., Dis.*, 2007.

[2] H.R. Cai, C.Y. Chung, and K.P. Wong, "Application of differential evolution algorithm for transient stability constrained optimal power flow," *IEEE Trans. Power Syst.*, 2008.

Computation flowchart



Our contributions to this field (with acknowledgement of funding providers)

2012-14

[1] Review and classification of TSCOPF

[2] Hybrid method for TSCOPF

[3,4] Data-driven methods

2015

[5] TSCUC (decomposition method)

[6] OMIB-based preventive control

2017

[7] Robust TSCOPF considering uncertain dynamic loads

2018

[8] Robust TSCOPF considering uncertain wind power

2020

[9] Direct method for preventive transient stability control for wind power variation

[10] Preventive-corrective coordinated TSCOPF with uncertain wind power

2022-23

[11] Fully robust Preventive-corrective coordinated TSCOPF

[12] Frequency-constrained optimal load restoration

[13] Data-driven method for stability-constrained load restoration

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Our contributions to this field

1. **Y. Xu,** Z.Y. Dong, Z. Xu, R. Zhang, and K.P. Wong, “Power system transient stability-constrained optimal power flow: a comprehensive review,” *Proc. IEEE PES General Meeting*, San Diego, 2012.
2. **Y. Xu**, Z.Y. Dong, K. Meng, J.H. Zhao, and K.P. Wong, “A hybrid method for transient stability constrained-optimal power flow computation,” *IEEE Trans. Power Systems*, 2012. – the best TSCOPF results on New England 30-bus system in the literature as of 2012.
3. **Y. Xu**, Z.Y. Dong, L. Guan, R. Zhang, K.P. Wong, and F. Luo, “Preventive dynamic security control of power systems based on pattern discovery technique,” *IEEE Trans. Power Systems*, 2012.
4. **Y. Xu**, Z.Y. Dong, R. Zhang, and K.P. Wong, “A decision tree-based on-line preventive control strategy for power system transient instability prevention,” *International Journal of Systems Science*, 2014.
5. **Y. Xu**, Z.Y. Dong, R. Zhang, Y. Xue, and D.J. Hill, “A decomposition-based practical approach to transient stability-constrained unit commitment,” *IEEE Trans. Power Systems*, 2015. – the 2nd paper for TSCUC.
6. **Y. Xu**, Z.Y. Dong, J. Zhao, Y. Xue, and D.J. Hill, “Trajectory sensitivity analysis on the equivalent OMIB of multi-machine systems for preventive transient stability control,” *IET Gen. Trans. & Dist.*, 2015.
7. **Y. Xu**, J. Ma, Z.Y. Dong, and D.J. Hill, “Robust transient stability-constrained optimal power flow with uncertain dynamic loads,” *IEEE Trans. Smart Grid*, 2017. – the 1st paper for TSCOPF with load dynamics and uncertainty.
8. **Y. Xu**, M. Yin, Z.Y. Dong, R. Zhang, and D.J. Hill, “Robust dispatch of high wind power-penetrated power systems against transient instability,” *IEEE Trans. Power Syst.*, 2018. – the 1st paper for TSCOPF with wind power uncertainty.
9. H. Yuan, **Y. Xu**, “Trajectory Sensitivity based Preventive Transient Stability Control of Power Systems against Wind Power Variation,” *Int. J. Electrical Power and Energy Systems*, 2020.
10. H. Yuan, **Y. Xu**, **“**Preventive-Corrective Coordinated Transient Stability Dispatch of Power Systems with Uncertain Wind Power,” *IEEE Trans. Power Syst.*, 2020.
11. H. Yuan, **Y. Xu**, and C. Zhang, “Robustly Coordinated Generation Dispatch and Load Shedding for Power Systems against Transient Instability under Uncertain Wind Power,” *IEEE Trans. Power Syst.*, 2022. – the 1st truly robust optimization method for TSCOPF with wind power uncertainty.
12. D. Xie, **Y. Xu**, S. Nadarajan, V. Viswanathan, and A.K. Gupta, “Dynamic Frequency-Constrained Load Restoration Considering Multi-Phase Cold Load Pickup Behaviours,” *IEEE Trans. Power Syst.*, 2023.
13. D. Xie, **Y. Xu**, S. Nadarajan, V. Viswanathan, and A.K. Gupta, “A Transparent Data-Driven Method for Stability-Constrained Load Restoration Considering Multi-Phase Load Dynamics,” *IEEE Trans. Power Syst.*, 2023.

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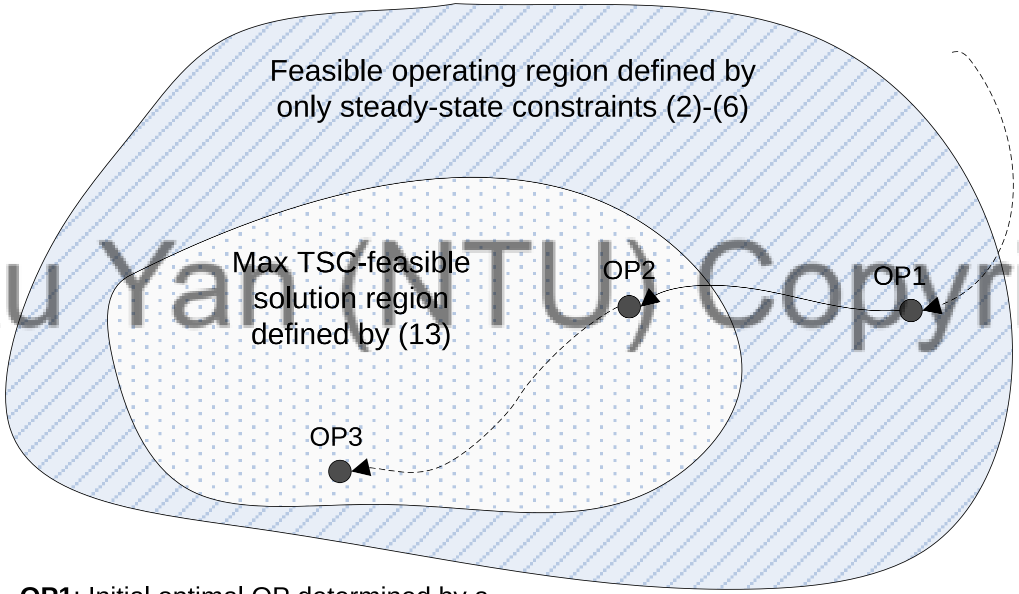
Hybrid computation method for TSCOPF [2]

- **Motivation:** combining classical programming and EA-enhanced stochastic search.
- **Key idea:** rather than coding all the control variables in the EA, only the maximum P_G output (TSC-feasible region) are searched by EA, and all the OPF variables are optimized by the interior-point (IP) method → global optimality, no limits on TSI (including multi-swing stability), TSA tool, system model, significantly enhanced evolutionary speed.



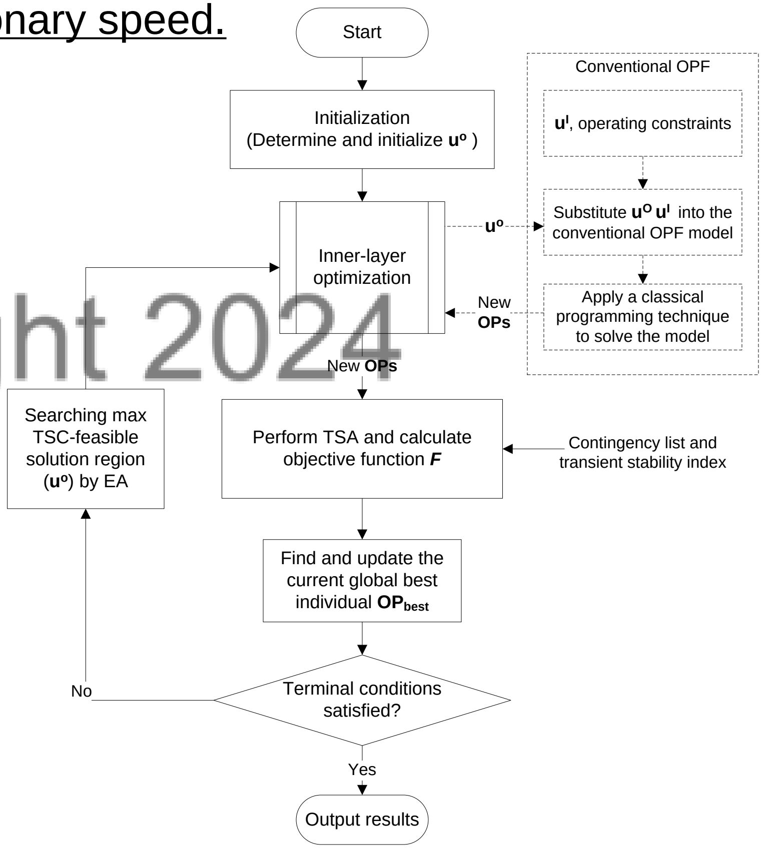
OP1: Initial optimal OP determined by a conventional OPF

OP2: OP located in the max TSC-feasible solution region

OP3: Global optimal OP

→ EA optimization

⇢ Conventional OPF calculation



- **Totally computing time**: maximum generation number × population size × (OPF solving time + TSA computing time)

→ compared with existing EA-based method, its population size has been reduced by at least 50%.

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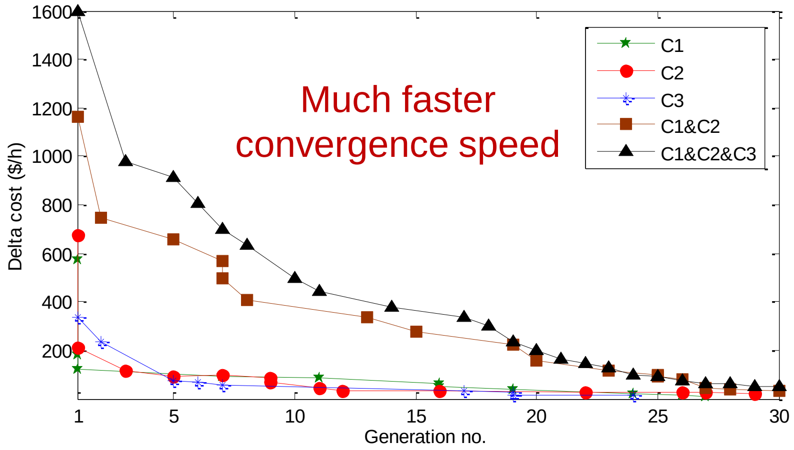
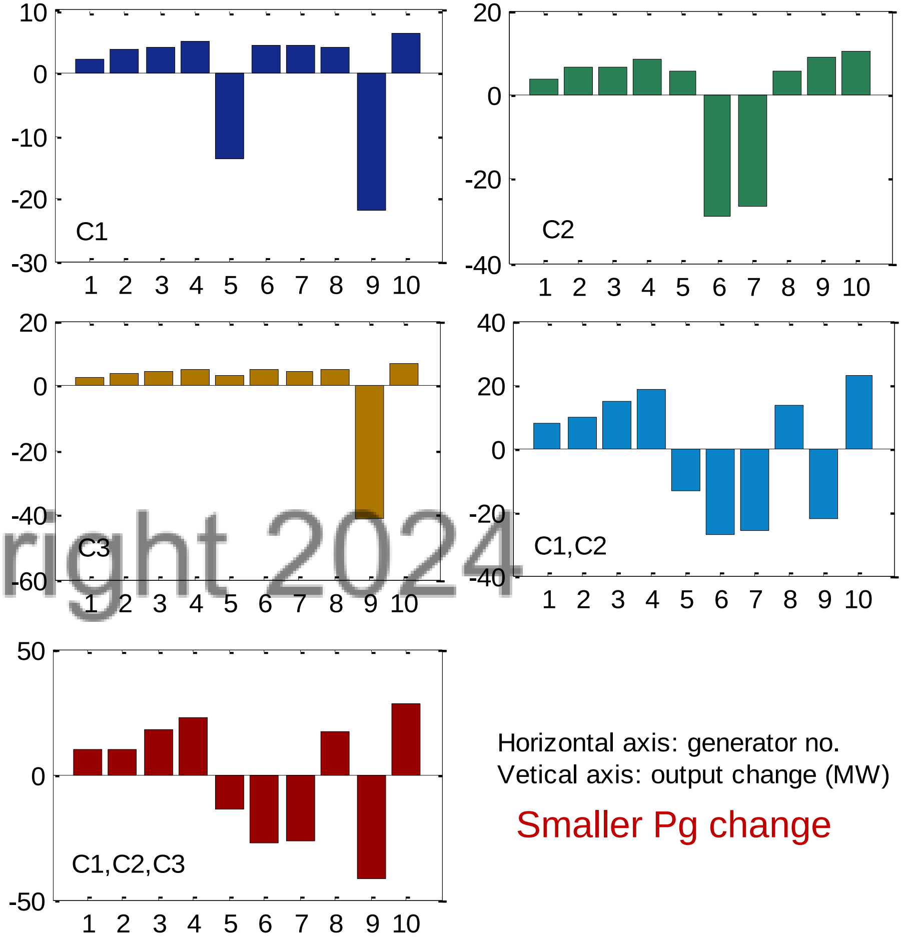
Hybrid computation method for TSCOPF [2]



TSC-OPF CALCULATION RESULTS OF NEW ENGLAND SYSTEM

Contingency	C1	C2	C3	C1, C2	C1,C2,C3
G1 (MW)	244.89	246.32	245.03	250.65	252.59
G2 (MW)	570.20	572.73	570.40	576.19	576.20
G3 (MW)	646.21	648.91	646.41	656.79	660.33
G4 (MW)	634.96	638.00	634.70	647.97	652.12
G5 (MW)	494.50	513.70	511.45	494.50	494.50
G6 (MW)	655.13	621.50	655.31	623.50	623.50
G7 (MW)	562.53	531.50	562.70	532.00	532.00
G8 (MW)	539.46	541.01	540.21	548.92	552.57
G9 (MW)	807.50	838.21	788.40	807.50	788.00
G10 (MW)	983.38	987.56	983.76	1000.02	1005.59
Stability margin	8.0	9.6	9.8	9.4 (C1) 7.8 (C2)	4.5 (C1) 9.7 (C2) 8.9 (C3)
△Cost ($/h)	7.78	19.83	15.29	34.28	50.86
△Cost ($/h) in the literature	833.65[11] 84.53[17] 16.16[14]	1190.0[10] 51.74[17] 19.19[14]	176[10]	1566.72[11]	Never reported



Horizontal axis: generator no.
Vetical axis: output change (MW)

Smaller Pg change

The best TSCOPF simulation results on New England 39-bus system as of year 2012 (to check if the results are beaten as of now)

[2] Y. Xu, Z.Y. Dong, K. Meng, J.H. Zhao, and K.P. Wong, "A hybrid method for transient stability constrained-optimal power flow computation," *IEEE Trans. Power Systems*, 2012.

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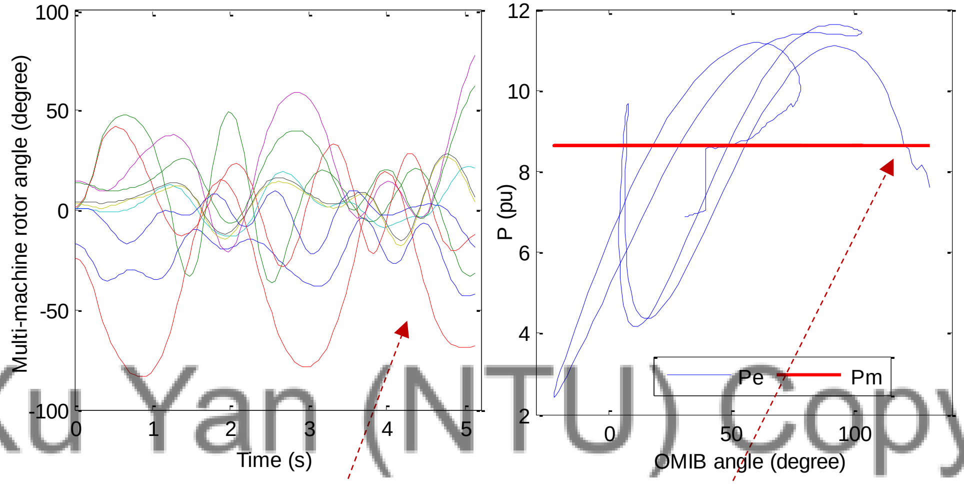
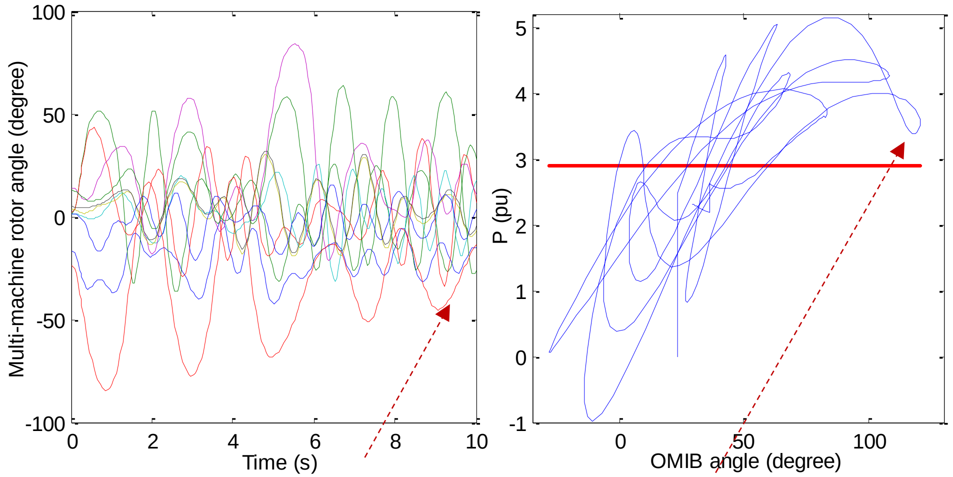
Hybrid computation method for TSCOPF [2]

Simulation results on New England 39-bus system (multi-swing stability case)

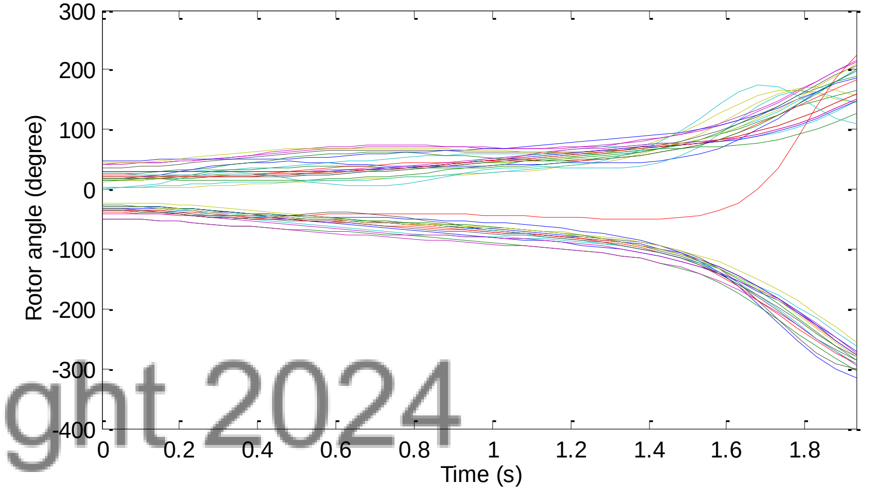


Base case: first-swing stable but multi-swing unstable

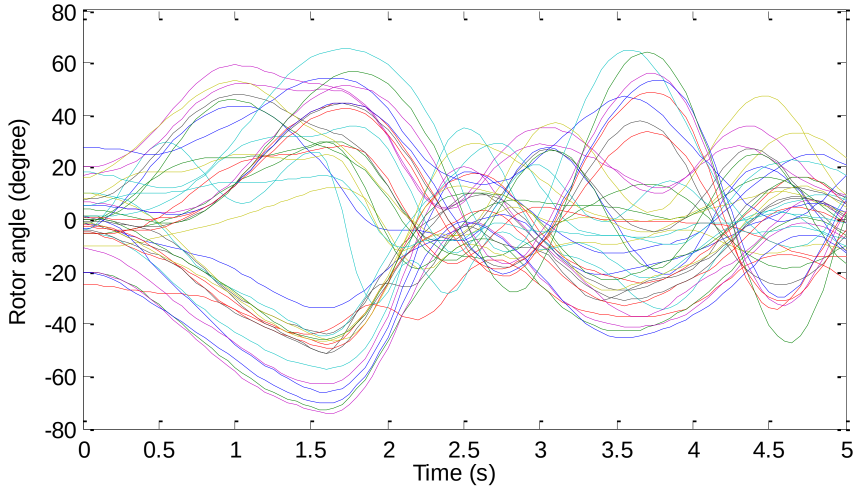


Solution result: multi-swing stable

Simulation results on a 39-gen 120-bus system (the equivalent model of a realistic power grid)



Base case: unstable



Solution result: stable

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Definition & classification
Challenges
Assessment & Control
Optimization Model

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Evolutionary Algorithm

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Full Robust TSCOPF

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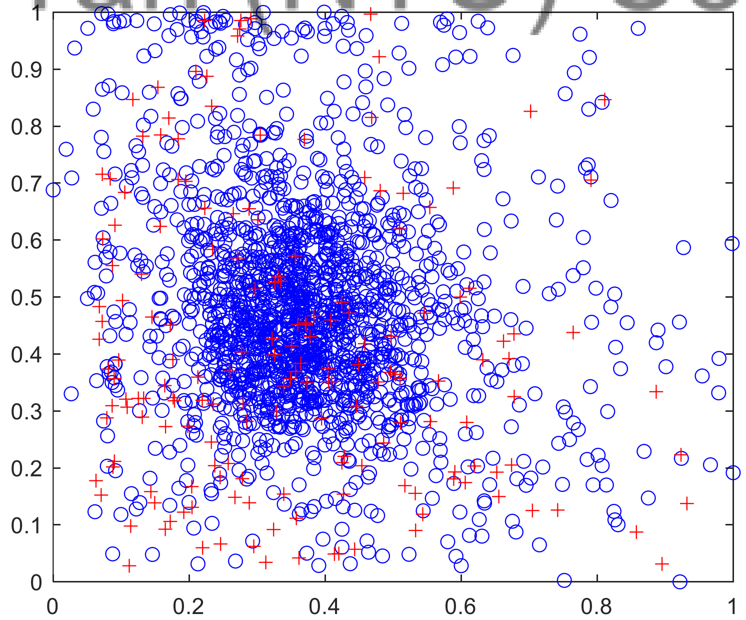
Pattern discovery-based data-driven method for TSCOPF [3]

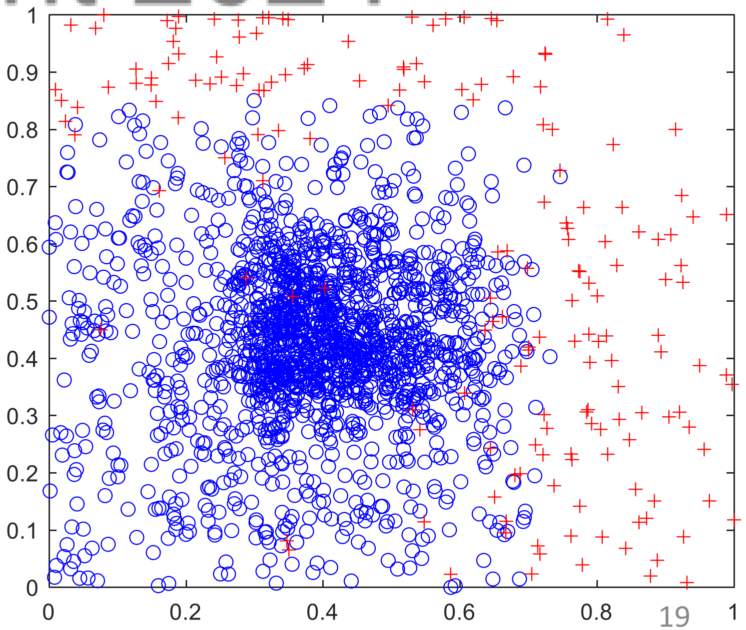
- **Critical generators identification** → feature estimation and selection (based on features' importance on system stability status)
- **Relief method:** evaluate the quality of features according to how well their values distinguish among instances near each other; Consider both the difference in features' values and classes, as well as the distance between the instances; Good features can cluster similar instances and separate dissimilar ones in the distance space.

$$diff(X,R,R') = \frac{|value(X,R) - value(X,R')|}{\max(X) - \min(X)}$$

$$W[X]^{i+1} = W[X]^i - \sum_{j=1}^{k} diff(X,R_i,H_j)/(m\cdot k) + \sum_{C\neq class(R_i)}^{k} [\frac{P(C)}{1-P(class(R_i))} \cdot \sum_{j=1}^{k} diff(X,R_i,M_j(C))]/(m\cdot k)$$



Non-significant features mix the instances



Significant features well distinguish the instances

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Pattern discovery-based data-driven method for TSCOPF [3]

- **Pattern discovery (PD):** search all the significant events in the instance space.
- **Residual analysis:** the difference between an event's observed (actual) occurrence probability and expected occurrence probability.
- **Recursively partitioning:** divide the instance space with residual evaluation of each hyper-rectangle, until all the significant events (patterns) are identified.

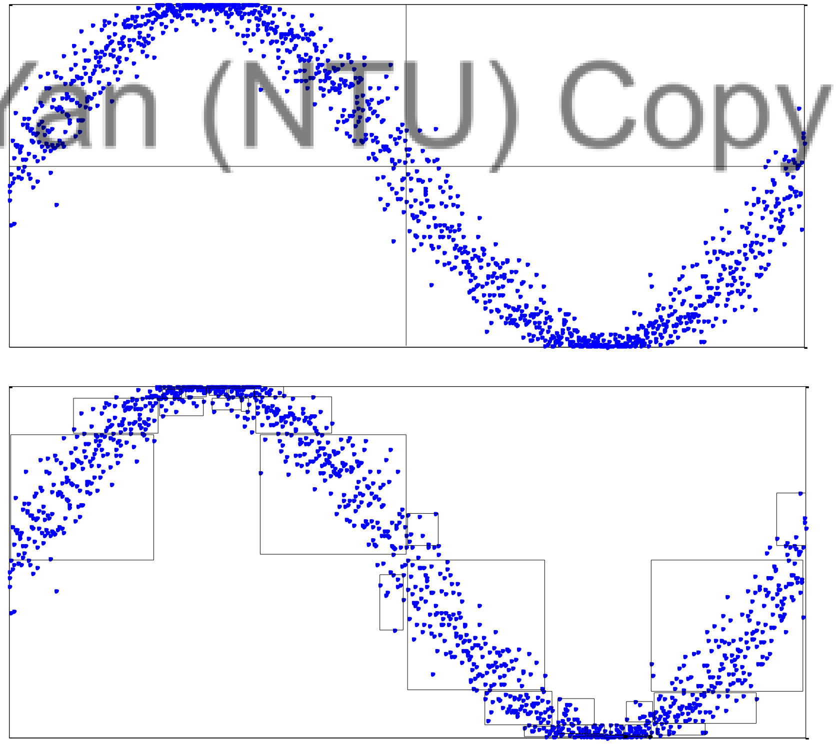


Illustration of PD by residual analysis and recursive partitioning.

Key definitions of PD:

Consider a continuous data set Ω in the N-dimensional *Euclidean* space $\Re^N$, let $\mathbf{X} = \{X_1, X_2, \ldots, X_N\}$ represent its feature set, and each feature X_i, $1 \le i \le N$, takes on values from its domain d_i, $d_i \subset \Re$. The following definitions are made for PD [19], [20].

Event: an event, E, is a *Borel* subset [28] of $\Re^N$, while a *Borel* subset geometrically forms an N-dimensional hyper-rectangle in $\Re^N$, defined by

$$E = I_1 \times I_2 \times \cdots \times I_N = \{\mathbf{X} : X_i \in I_i,\ 1 \le i \le N\} \quad (1)$$

where $I_i = (a_i, b_i]$ is a one-dimensional semi-closed interval along the ith feature, $-\infty < a_i < b_i < \infty$.

Volume: the volume of an event, υ, is the hyper-volume occupied by the *Borel* subset. Let L_i represent the length of the ith interval I_i of event E, $L_i = |b_i - a_i|$, the volume of E is

$$\upsilon(E) = \prod_{i=1}^{N} L_i. \quad (2)$$

Observed frequency: the observed frequency of an event E, o_E, is the actual number of instances that fall inside the volume occupied by E.

Pattern: a pattern is a statistically significant event. Let $\vartheta(\cdot)$ be a test statistic corresponding to a specified discovery criterion c and θ_c^α be the critical value of the statistical test at a significant level of α. An event E is considered to be significant, i.e., a pattern, if it satisfies the condition

$$\vartheta(E) \ge \theta_c^\alpha. \quad (3)$$

Residual: as the statistic $\vartheta(\cdot)$ to test the significance of the pattern candidates [19], [20], the residual of an event E is the difference between its actual occurrence, i.e., observed frequency, and its expected occurrence:

$$\delta_E = o_E - e_E \quad (4)$$

where e_E is the expected occurrence, or expected frequency, under the pre-assumed model estimated by the given data set.

Pattern discovery-based data-driven method [3]

- Define the stability status of the "events":

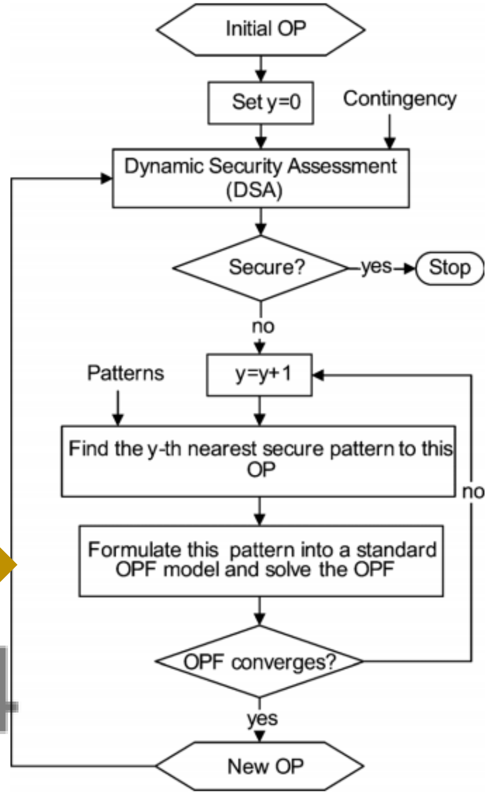
$$\begin{cases} \dfrac{M_S}{M_S + M_I} > \lambda \rightarrow \text{"secure"} \\ \dfrac{M_I}{M_S + M_I} \geq \lambda \rightarrow \text{"insecure"} \end{cases}$$

- Compose the statistically stable and unstable regions:

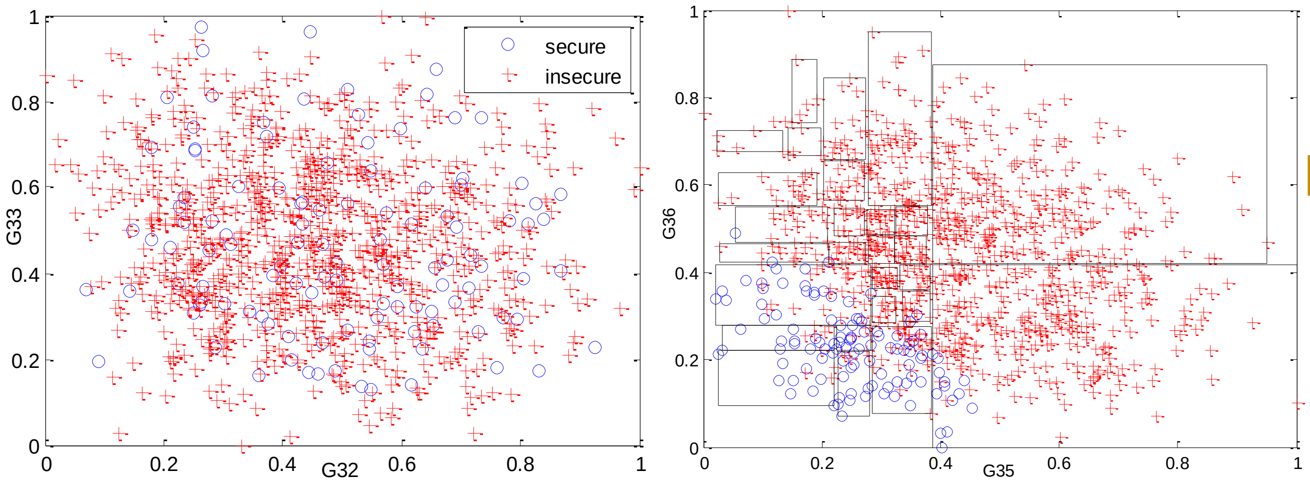
$$\mathbf{R}^S = \bigcup_{i=1}^{K} E_i^S \qquad \mathbf{R}^I = \bigcup_{j=1}^{J} E_j^I$$

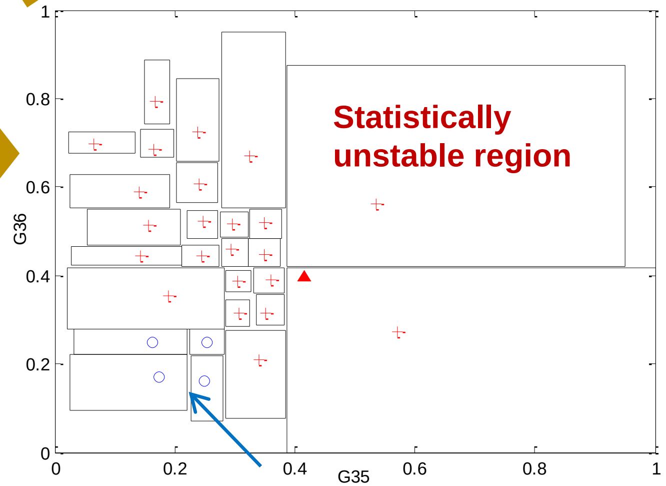
- Dispatch the unstable point to the nearest stable region:

$$D(R,R') = D(R',R) = \sqrt{\sum_{i=1}^{N} (X_i - X_i')^2}$$



Substitute the region boundaries into the OPF model as linear constraints





Statistically stable region

[3] Y. Xu, Z.Y. Dong, L. Guan, R. Zhang, K.P. Wong, and F. Luo, "Preventive dynamic security control of power systems based on pattern discovery technique," *IEEE Trans. Power Systems*, 2012.

1. Stability
Definition & classification
Challenges
Assessment & Control
Optimization Model

2. Review
Direct Method
Discretization Method
Data-Driven Method
Evolutionary Algorithm

3. Our Methods
Hybrid Method
Data-Driven Methods
TSC-Unit Commitment
Robust TSCOPF-Load
Robust TSCOPF-Wind
PC-CC Cor. TSCOPF
Full Robust TSCOPF

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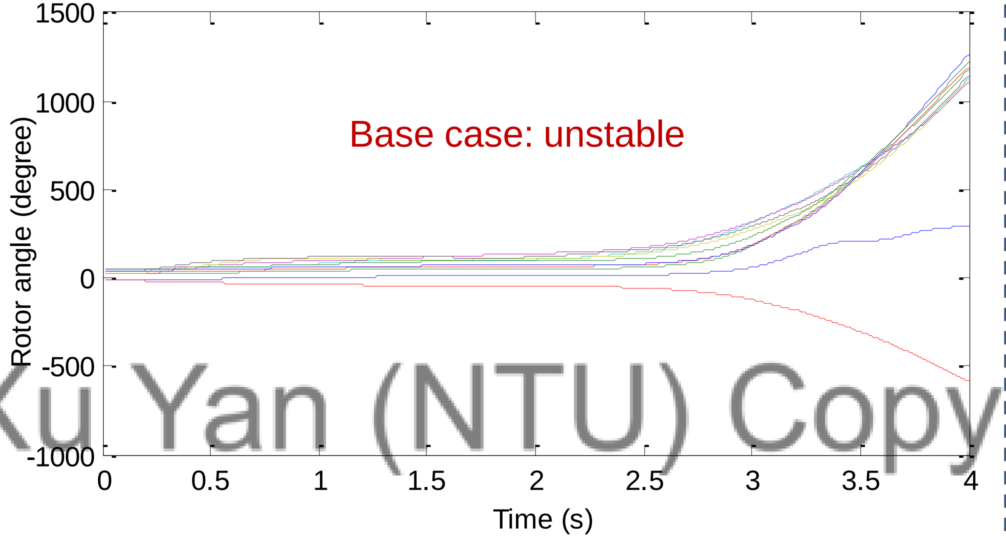
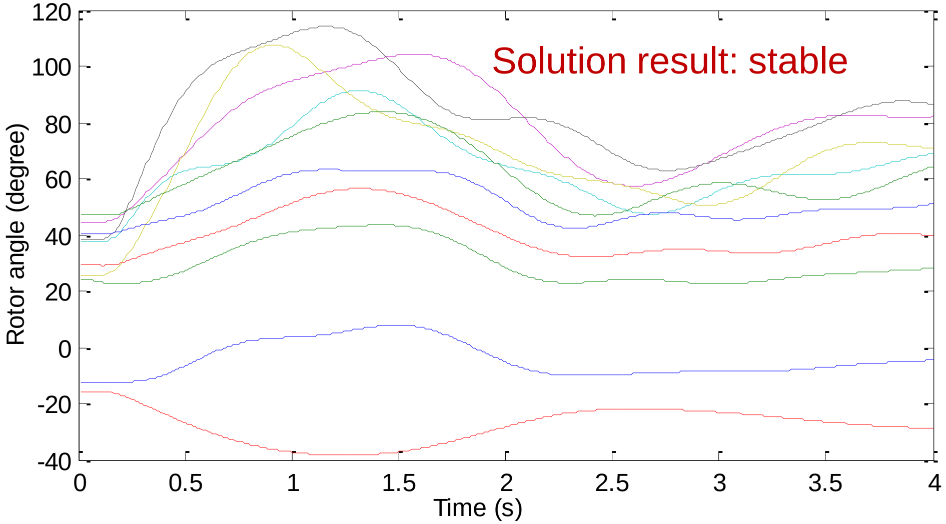
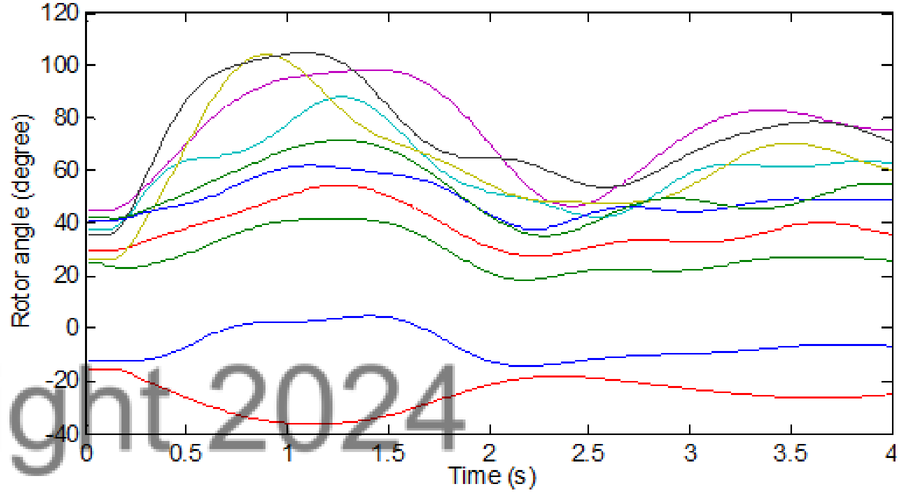
Pattern discovery-based data-driven method [3]

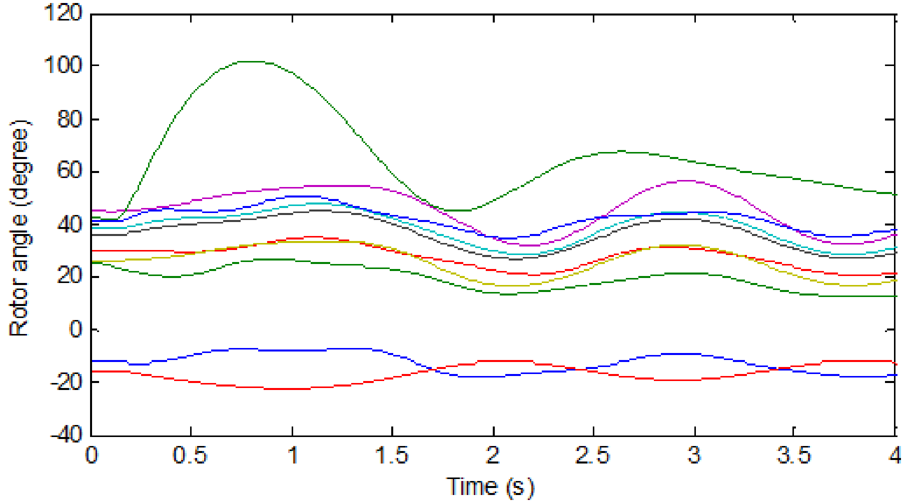
Simulation results on New England 39-bus system (single contingency case: fault 1)





Simulation results on New England 39-bus system (multi-contingency case: fault 1 & fault 2)





Solution results: all stable

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Decision tree-based data-driven method [4]

- **Decision tree (DT)-based stabilization rule:**

$$R_C = \{T \in S : N_i \le \eta_i, i \in \vartheta\}$$

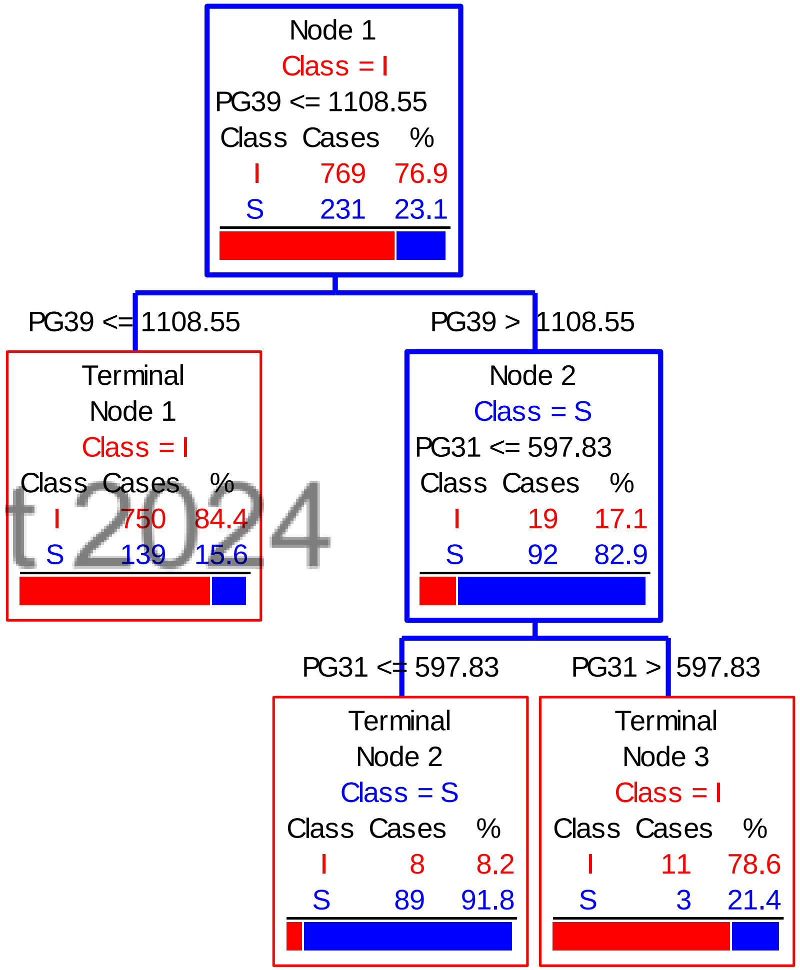
where R_C represents the tree growing for contingency C, T denotes the terminal nodes of a tree, S means “stable” class, N_i and η_i are the node i and the corresponding threshold, respectively, ϑ denotes the critical generator set.

- **Computation steps:**

1) Conduct TSA on current OP subject to contingency C, if it is “stable”, stop the computation; otherwise, go to step 2);
2) Modify tree splitting rules as $R_C = \{T \in S: N_i \le d_i \times \eta_i, i \in \vartheta\}$, and set $d_i = 1$;
3) Substitute the modified splitting rules into OPF model;
4) Solve the OPF, if it is convergent, obtain a new OP and go to step 5); otherwise, go to step 6);
5) Conduct TSA on the new OP subject to contingency C, if it is “stable”, stop the computation; otherwise, go to the next step;
6) Update $d_i = d_i + \varepsilon$ and return to step 3).

It should be emphasized that ε in step 6) is a user-defined parameter, it is used to increase the generation shifting in case the transient stability can’t be ensured from the original tree threshold η.

A trained DT from stability database of New England 39-bus system



Stabilization rule

$$R_{Fault1} = \{P_{G39} > 1108.55 \,\&\, P_{G31} \le 597.83\}$$

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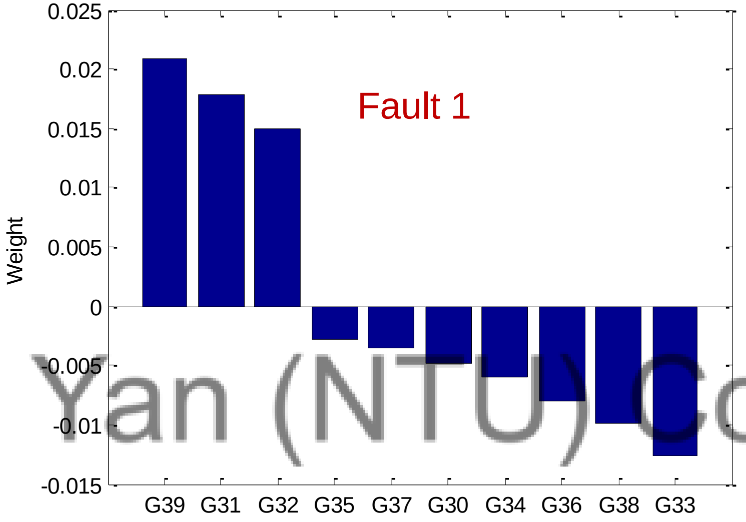
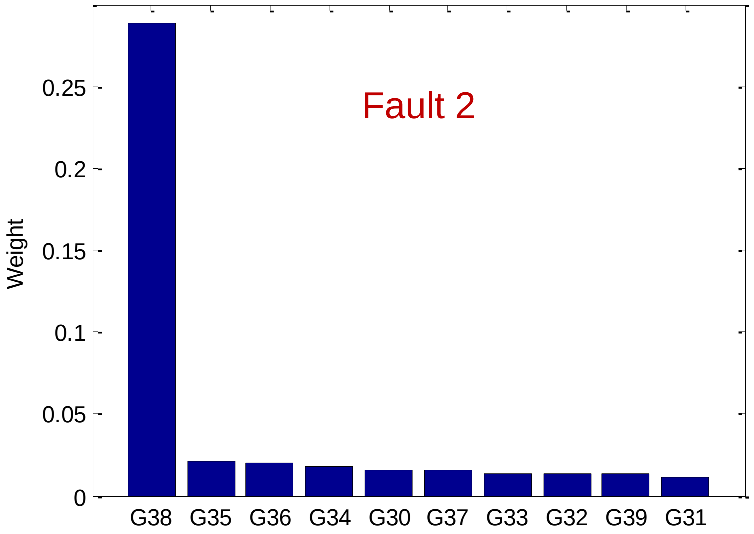
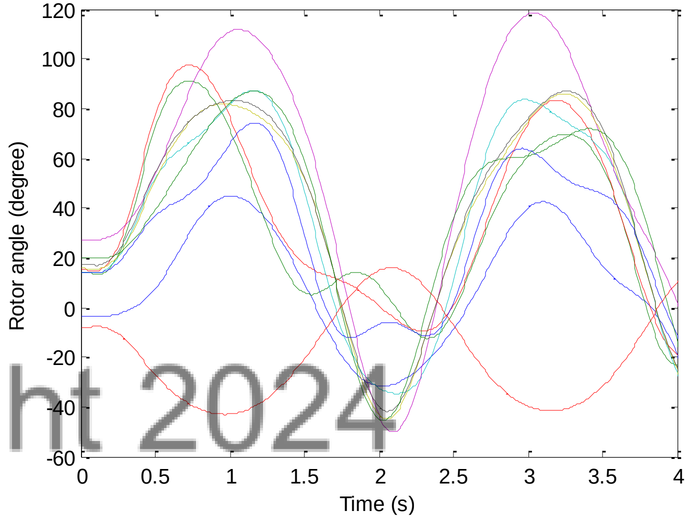
Decision tree-based data-driven method [4]

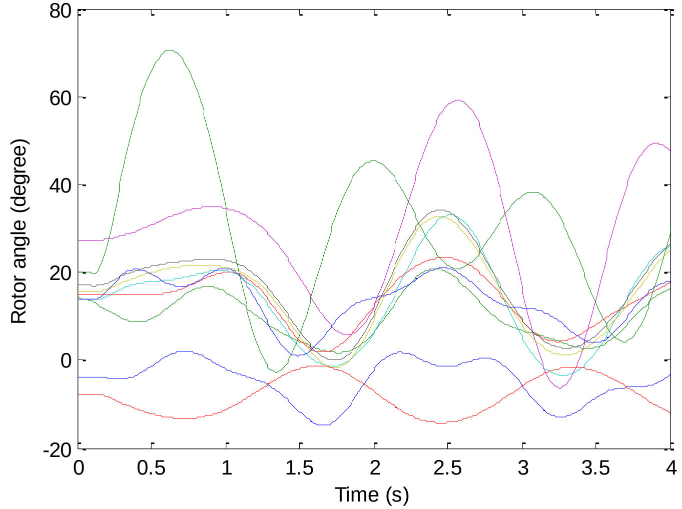
Simulation results on New England 39-bus system (critical feature selection)





Simulation results on New England 39-bus system (multi-contingency case: fault 1 & fault 2)





Solution results: all stable

Transient Stability-Constrained Unit Commitment (TSCUC) [5]

A. Objective Function

$$\min_{P,I} \sum_{t=1}^{NT}\sum_{i=1}^{NG}\left[C_i\left(P_{it}\right)\cdot I_{it} + CU_{it} + CD_{it}\right] \quad (1)$$

where $C_i\left(P_{it}\right)$ is the generation cost function:

$$C_i\left(P_{it}\right) = a_i \cdot I_{it} + b_i \cdot P_{it} + c_i \cdot \left(P_{it}\right)^2 \quad (2)$$

B. Operational Constraints

a) Power balance:

$$\sum_{i=1}^{NG} P_{it} \cdot I_{it} = D_t, \quad \forall t \quad (3)$$

b) Generation limits:

$$P_i^{\min} \cdot I_{it} \le P_{it} \le P_i^{\max} \cdot I_{it}, \quad \forall t \quad (4)$$

c) Spinning reserve limits:

$$\sum_{i=1}^{NG} R_{it} \cdot I_{it} \ge R_t^S, \quad \forall t \quad (5)$$

d) Ramping limits:

$$\begin{cases} RD_i \le P_{i(t+1)} - P_{it} \le RU_i \\ SD_i \le P_{i(t+1)} - P_{it} \le SU_i \end{cases}, \quad \forall t, \forall i \quad (6)$$

e) Minimum up and down time limits:

$$\begin{cases} \left(X_{i(t-1)}^{\text{on}} - T_i^{\text{on}}\right)\cdot\left(I_{i(t-1)} - I_{it}\right) \ge 0 \\ \left(X_{i(t-1)}^{\text{off}} - T_i^{\text{off}}\right)\cdot\left(I_{i(t-1)} - I_{it}\right) \ge 0 \end{cases}, \quad \forall t, \forall i \quad (7)$$

C. Steady-State Security Constraints

$$-F_l^{\max} \le F_{lt,k} = \sum_{i=1}^{NG} PTDF_{lt,i}^k \cdot P_{it} - \sum_{j=1}^{ND} PTDF_{lt,j}^k \cdot D_{jt} \le F_l^{\max}, \quad \forall t, \forall l, \forall k \quad (8)$$

where k=0 denotes the base case, and k=1,2…n denotes a contingency case, $PTDF_{lt,i}^k$ is the power transfer distribution factor of bus i to line l for contingency k at period t, D_{it} is the load demand of bus i.

D. Transient Stability Constraints

$$0 < \eta_{t,k} \le \varepsilon, \quad \forall t, \forall k \quad (9)$$

$\eta_{t,k}$ is calculated through a rigorous time-domain simulation-based TSA procedure.

- **Challenges:** a large-scale mixed-integer nonlinear programming (MINLP) model with DAE constraints.
- **Proposed method:** 1) decompose the problem into a master problem (UC) and a range of subproblems for steady-state security assessment and transient stability assessment (TSA); 2) generate Benders cut and stabilization cut to eliminate security/stability violations.

Extended Equal-Area Criterion (EEAC)

Critical Machines (CM) and Non-CMs (NM)

For CMs:

$$\begin{cases} \delta_C(t) = (M_C)^{-1} \cdot \sum_{i \in C} M_i \delta_i(t) \\ \omega_C(t) = (M_C)^{-1} \cdot \sum_{i \in C} M_i \omega_i(t) \end{cases}$$

For NMs:

$$\begin{cases} \delta_N(t) = (M_N)^{-1} \cdot \sum_{j \in N} M_j \delta_j(t) \\ \omega_N(t) = (M_N)^{-1} \cdot \sum_{j \in N} M_j \omega_j(t) \end{cases}$$

$$M_C = \sum_{r \in C} M_r \; ; \quad M_N = \sum_{q \in N} M_q$$

One-Machine-Infinite-Bus (OMIB) equivalent

$$\delta(t) = \delta_c(t) - \delta_N(t); \quad \omega(t) = \omega_C(t) - \omega_N(t)$$

$$P_m(t) = M \cdot \left[M_C^{-1} \cdot \sum_{r \in C} P_{mr}(t) - M_N^{-1} \cdot \sum_{q \in N} P_{mq}(t) \right]$$

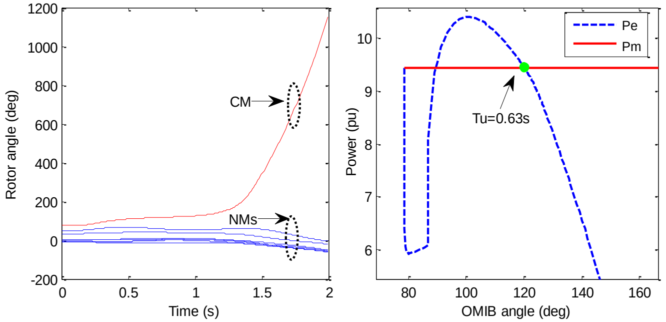
$$P_e(t) = M \cdot \left[M_C^{-1} \cdot \sum_{r \in C} P_{er}(t) - M_N^{-1} \cdot \sum_{q \in N} P_{eq}(t) \right]$$

$$M = (M_C \cdot M_N) \cdot (M_C + M_N)^{-1}$$

Transient Stability Assessment (TSA)

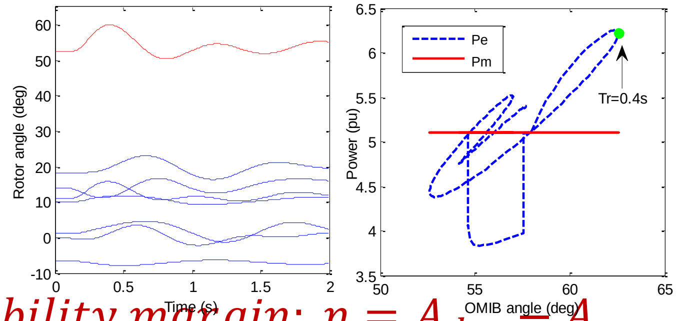
$$P_a(T_u) = P_m(T_u) - P_e(T_u) = 0, \quad \dot{P}_a(T_u) > 0$$

→ P_e crosses P_m *at* T_u (*time to instability*)



$$P_a(T_r) = P_m(T_r) - P_e(T_r) < 0, \quad \omega(T_r) = 0$$

→ P_e stops excursion and returns before crossing P_m *at* T_r (*time to first-swing stability*)



stability margin: $\eta = A_{dec} - A_{acc}$

$$= \begin{cases} -M\left(\omega(T_u)\right)^2/2, & \textit{unstable} \\ |P_a(T_r)|\left(\delta(T_u) - \delta(T_r)\right)/2, & \textit{stable} \end{cases}$$

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Stabilization Cut Derivation [5]

Working Principle

Stabilizing an unstable system consists of modifying the pre-contingency conditions until the stability margin η becomes zero (or positive). This can be achieved by increasing the decelerating area A_{dec} and/or decreasing the accelerating area A_{acc} of the OMIB $P-\delta$ representation. In practice, this can be realized by decreasing the OMIB mechanical power $P_m(t_0)$:

$$\Delta P_m(t_0) = M\cdot(M_C)^{-1}\cdot\Delta P_C - M\cdot(M_N)^{-1}\cdot\Delta P_N \qquad (21)$$

where t_0 denotes the pre-contingency state, ΔP_C and ΔP_N are respectively the changes in the total power of CMs and NMs:

$$\Delta P_C = \sum_{i\in C}\Delta P_{mi}(t_0);\quad \Delta P_N = \sum_{j\in N}\Delta P_{mj}(t_0) \qquad (22)$$

To maintain the power balance, the following condition should be satisfied:

$$\Delta P_N = -\Delta P_C \qquad (23)$$

Substituting (23) into (21), we have:

$$\begin{aligned}\Delta P_m(t_0) &= \left[M\cdot(M_C)^{-1} + M\cdot(M_N)^{-1}\right]\cdot\Delta P_C\\ &= -\left[M\cdot(M_C)^{-1} + M\cdot(M_N)^{-1}\right]\cdot\Delta P_N\end{aligned} \qquad (24)$$

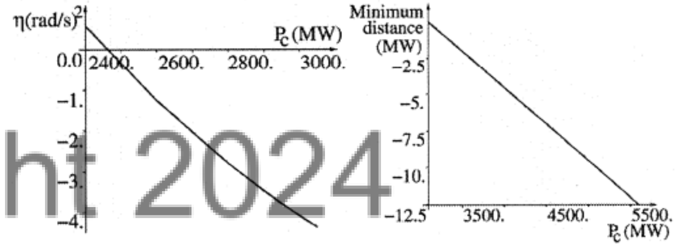
Eq. (21)-(24) reveal that by shifting real power output of CMs to NMs, the transient stability can be restored.

Proposed Stabilization Cut

Numerous examples have reported a quasi-linear relationship between changes of stability margin and OMIB mechanical power at pre-contingency state:

$$\Delta\eta = \varsigma\cdot\Delta P_m(t_0) \qquad (25)$$

where ς is the approximate linear sensitivity of the stability margin with respect to generation change.



In practice, the sensitivity value around the operating point n can be numerically estimated via two successive EEAC runs:

$$\varsigma_n = \left[\Delta\eta_{(n-2)} - \Delta\eta_{(n-1)}\right] / \left[\Delta P_m(t_0)_{(n-2)} - \Delta P_m(t_0)_{(n-1)}\right] \qquad (26)$$

With ς_n , the required generation shifting for TSC can be analytically calculated. Specifically, to control an unstable case, whose stability margin is η_{us} ($\eta_{us} < 0$), if the desired stability margin is ε ($\varepsilon \ge 0$), the required increment in stability margin is $\Delta\eta \ge -\eta_{us} + \varepsilon$. Combining (24)-(26), the required generation shifting between CMs and NMs can be calculated as:

$$\Delta P_C \ge \frac{-\eta_{us}+\varepsilon}{\varsigma_n}\cdot\left[M\cdot(M_C)^{-1} + M\cdot(M_N)^{-1}\right]^{-1} \qquad (27)$$

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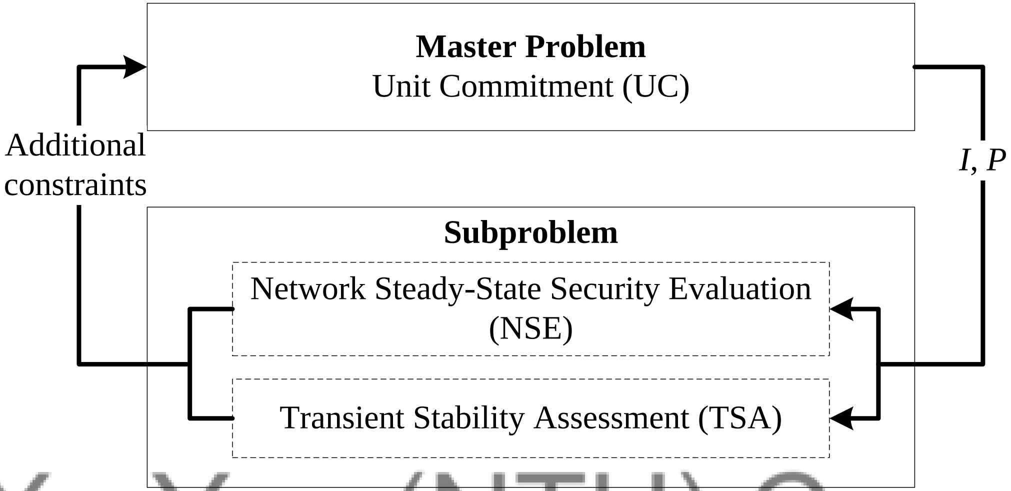
Transient Stability-Constrained Unit Commitment (TSCUC) [5]



Proposed Decomposition Framework

Let $\mathbf{x}$ denote the UC status I and generation dispatch P, and $\mathbf{y}$ denote the system state variables. The TSCUC problem can be rewritten as the following standard BD form:

$$\min \quad \mathbf{c}^T\mathbf{x} \tag{28}$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{x} \geq \mathbf{b} \tag{29}$$

$$\mathbf{E}\mathbf{x} + \mathbf{F}\mathbf{y} \geq \mathbf{h} \tag{30}$$

$$\mathbf{0} \leq \boldsymbol{\eta} \leq \boldsymbol{\varepsilon} \tag{31}$$

where (28) corresponds to the cost function (1), (29) corresponds to the operational constraints (3)-(7) as well as the additional constraints generated from the subproblem, (30) corresponds to the network steady-state security constraints (8), and (31) corresponds to the transient stability constraints (9).

Network Steady-State Security Evaluation (NSE)

The NSE involves both the base case and contingency cases. For each case, a linear programming (LP) model is built [3]:

$$\min \quad \nu(\hat{\mathbf{x}}) = \mathbf{1}^T\mathbf{s} \tag{32}$$

$$\text{s.t.} \quad \mathbf{F}\mathbf{y} + \mathbf{s} \geq \mathbf{h} - \mathbf{E}\hat{\mathbf{x}}, \qquad \boldsymbol{\pi} \tag{33}$$

where $\mathbf{1}$ is the vector of ones, $\mathbf{s}$ is the slack vector used to check the violation of line flow constraints, and $\boldsymbol{\pi}$ is the *Lagrangian* multiplier vector of inequality constraints in (33). $\nu(\hat{\mathbf{x}}) > 0$ means the violation occurs, and the Benders cut is generated as:

$$\nu(\mathbf{x}) = \nu(\hat{\mathbf{x}}) - \boldsymbol{\pi}^T\mathbf{E}(\mathbf{x} - \hat{\mathbf{x}}) \leq 0 \tag{34}$$

$\boldsymbol{\pi}^T\mathbf{E}$ mathematically represents the marginal decrement or increment of the objective function (32) when $\mathbf{x}$ is adjusted. In the next iteration, (34) will be added to (29) of the master problem to eliminate the steady-state security violation.

Stabilization cut

$$\sum_{i\in C} P_{mi}(t_0) - \sum_{i\in C} \hat{P}_{mi}(t_0) \geq \frac{-\eta_{us} + \varepsilon}{\varsigma_n} \cdot \left[M \cdot (M_C)^{-1} + M \cdot (M_N)^{-1} \right]^{-1} \tag{35}$$

where $\hat{P}_{mi}(t_0)$ denotes the generation output of unit i obtained from the master problem.

Linear algebraic form

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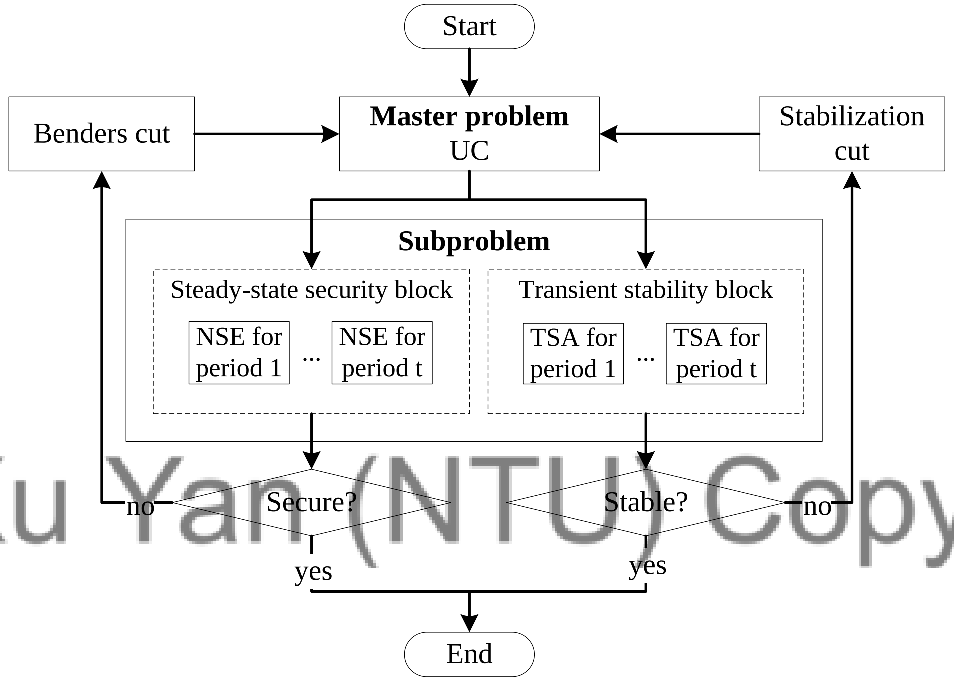
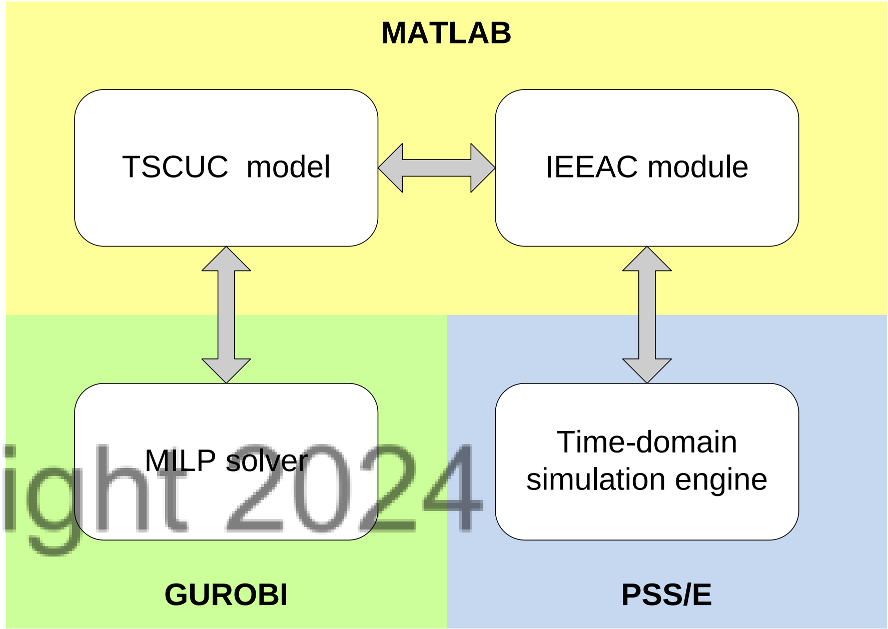
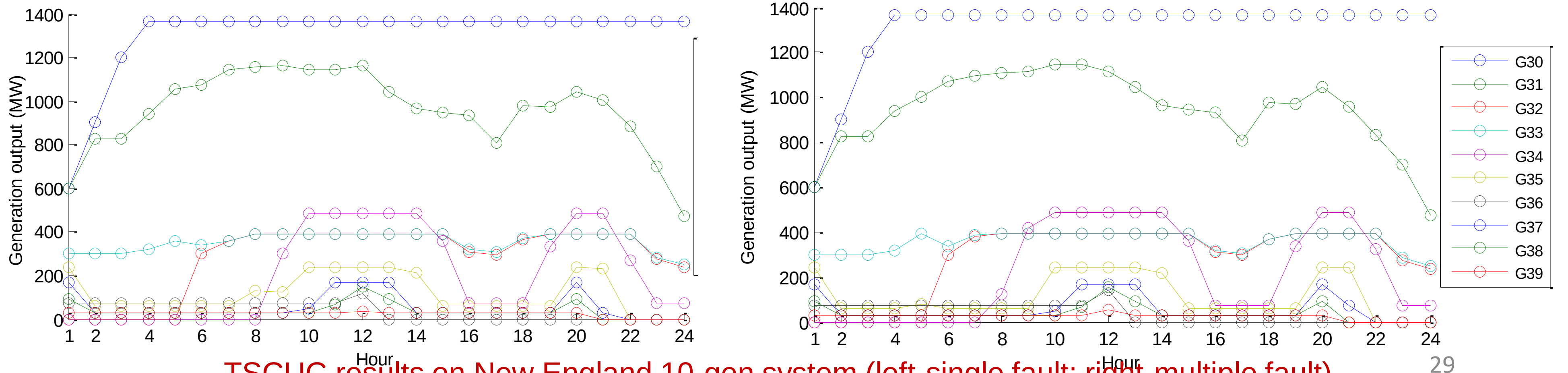
Transient Stability-Constrained Unit Commitment (TSCUC) [5]

Computation Flowchart



Implementation structure





TSCUC results on New England 10-gen system (left-single fault; right-multiple fault)

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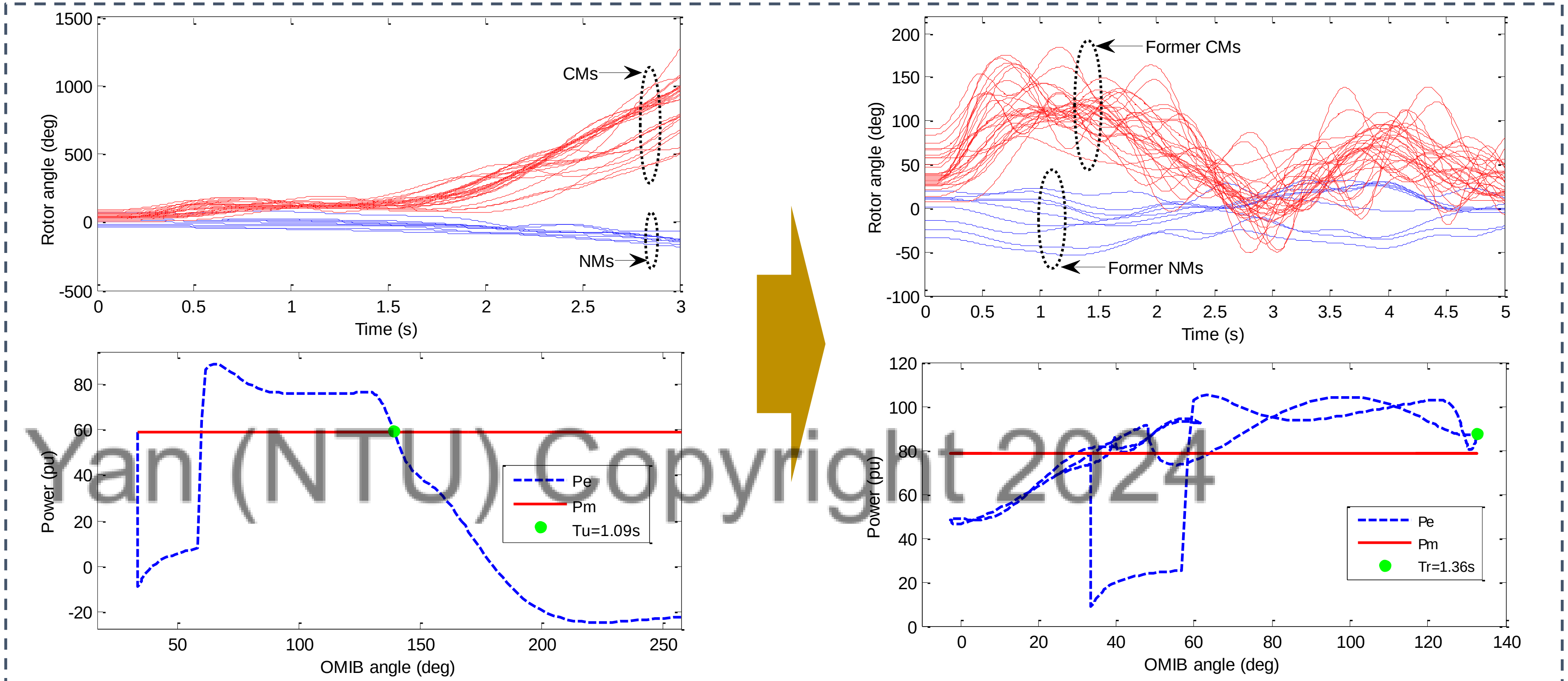
Transient Stability-Constrained Unit Commitment (TSCUC) [5]



Rotor angles of IEEE 50-gen system (left: base case; right: solution results)

CPU TIME (IEEE 50-MACHINE SYSTEM)

UC by MILP (T_{UC})	NSE for BD cut (T_{NSE})	TSA by EEAC+PSS/E (T_{TDS})
13.7s	0.3s	2.2s~2.7s

TOTAL CPU TIME FOR TSCUC COMPUTATIONS

50-machine 145-bus system (method of this paper)	54-machine 118-bus system (method of [5])	69-machine 300-bus system (method of [5])
319s	45015s	56305s

[5] Y. Xu, Z.Y. Dong, R. Zhang, Y. Xue, and D.J. Hill, "A decomposition-based practical approach to transient stability-constrained unit commitment," *IEEE Trans. Power Systems*, 2015. – the 2nd paper for TSCUC, 140 times faster than the first paper in the literature.

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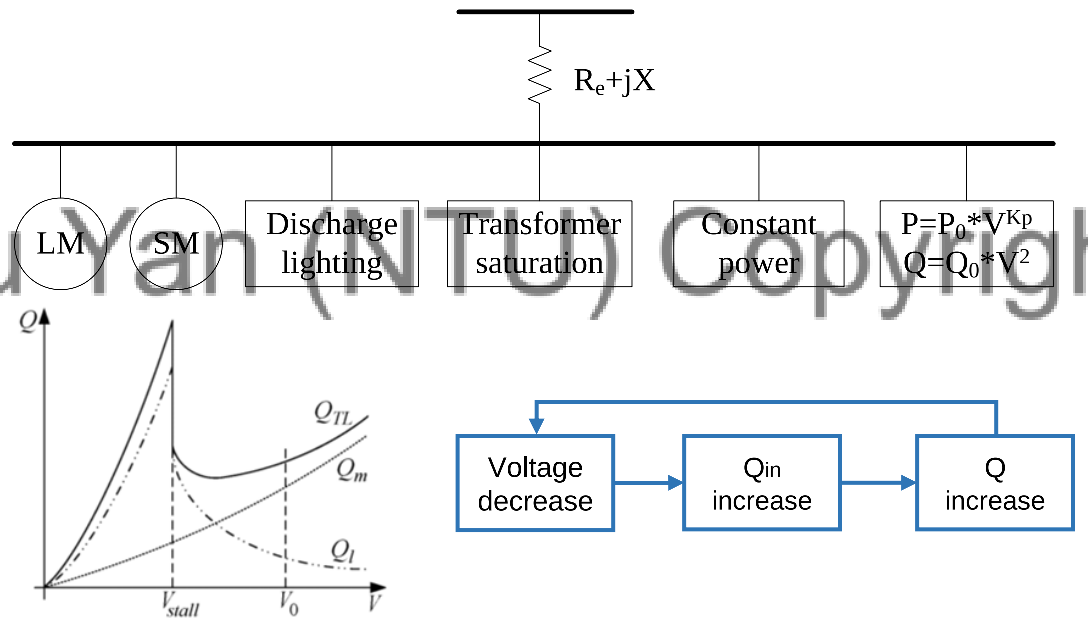
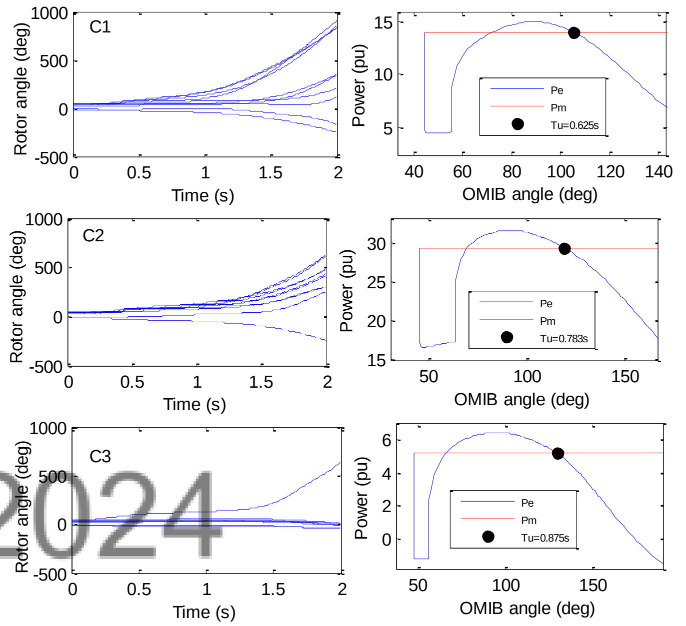
Robust TSC-OPF with uncertain Dynamic Loads [7]

- **Problem descriptions:**

1) Load dynamics has a substantial impact on transient stability but has not been properly treated in TSC-OPF problems → all the conventional works only consider static loads.

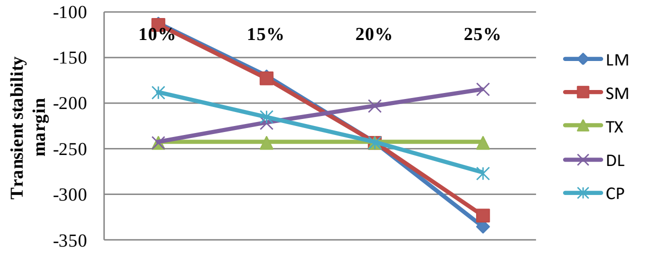
Composite Load Components & Load Dynamic Behavior





Considering dynamic load models, all the stable TSCOPF solutions become unstable [R]

2) Load compositions are very difficult, if not impossible, to estimate for online TSC-OPF calculation, and their variations have a significant impact on transient stability.



Load component (%) V.S. stability margin [R]

[R] R. Zhang, Y. Xu, W. Zhang, et al, "Impact of dynamic load models on transient stability-constrained optimal power flow," *Proc. IEEE APPEEC Conference, 2016.*

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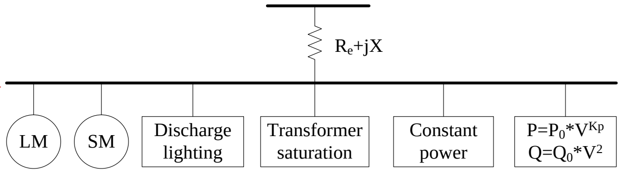
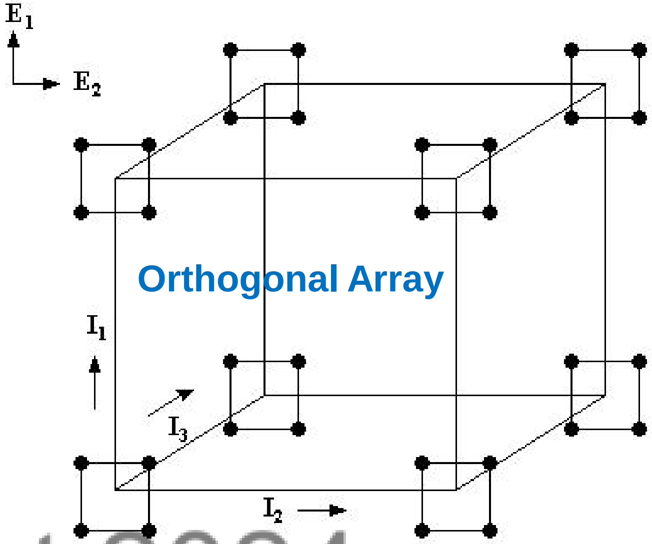
Robust TSC-OPF with uncertain Dynamic Loads [7]

- **Augmented TSC-OPF modelling:**

$$\min_{\mathbf{u}} \ f(\mathbf{x},\mathbf{u})$$
$$\text{s.t.} \ \mathbf{g}(\mathbf{x},\mathbf{u}) = 0$$
$$\mathbf{h}(\mathbf{x},\mathbf{u}) \le 0$$
$$TSI(\mathbf{x},\mathbf{u},\tilde{\varsigma}) \ge \varepsilon$$

u to make the system ***robust stable*** to the random variations of load model $\tilde{\varsigma}$





- **Key challenges:**

1) How to model the uncertain parameters in TSC-OPF?
2) How to efficiently solve the uncertain TSC-OPF model?

- **Proposed approach:**

1) Robust design based on Taguchi's Orthogonal Array Testing (TOAT) for uncertainty modeling → to select a small number of testing scenarios with good statistical information in the uncertainty space.
2) Trajectory sensitivity-based critical uncertain parameters identification → no need to model all load parameters, hence smaller problem size.
3) Decomposition-based solution framework → high efficiency
4) EEAC-based stabilization cut construction

TABLE I TESTING SCENARIOS DETERMINED OF OA $L_8(2^7)$

Testing scenario	Variable levels						
	$\tilde{\varsigma}_1$	$\tilde{\varsigma}_2$	$\tilde{\varsigma}_3$	$\tilde{\varsigma}_4$	$\tilde{\varsigma}_5$	$\tilde{\varsigma}_6$	$\tilde{\varsigma}_7$
1	$\varsigma_1(1)$	$\varsigma_2(1)$	$\varsigma_3(1)$	$\varsigma_4(1)$	$\varsigma_5(1)$	$\varsigma_6(1)$	$\varsigma_7(1)$
2	$\varsigma_1(1)$	$\varsigma_2(1)$	$\varsigma_3(1)$	$\varsigma_4(2)$	$\varsigma_5(2)$	$\varsigma_6(2)$	$\varsigma_7(2)$
3	$\varsigma_1(1)$	$\varsigma_2(2)$	$\varsigma_3(2)$	$\varsigma_4(1)$	$\varsigma_5(1)$	$\varsigma_6(2)$	$\varsigma_7(2)$
4	$\varsigma_1(1)$	$\varsigma_2(2)$	$\varsigma_3(2)$	$\varsigma_4(2)$	$\varsigma_5(2)$	$\varsigma_6(1)$	$\varsigma_7(1)$
5	$\varsigma_1(2)$	$\varsigma_2(1)$	$\varsigma_3(2)$	$\varsigma_4(1)$	$\varsigma_5(2)$	$\varsigma_6(1)$	$\varsigma_7(2)$
6	$\varsigma_1(2)$	$\varsigma_2(1)$	$\varsigma_3(2)$	$\varsigma_4(2)$	$\varsigma_5(1)$	$\varsigma_6(2)$	$\varsigma_7(1)$
7	$\varsigma_1(2)$	$\varsigma_2(2)$	$\varsigma_3(1)$	$\varsigma_4(1)$	$\varsigma_5(2)$	$\varsigma_6(2)$	$\varsigma_7(1)$
8	$\varsigma_1(2)$	$\varsigma_2(2)$	$\varsigma_3(1)$	$\varsigma_4(2)$	$\varsigma_5(1)$	$\varsigma_6(1)$	$\varsigma_7(2)$

[7] Y. Xu, J. Ma, Z.Y. Dong, and D.J. Hill, "Robust transient stability-constrained optimal power flow with uncertain dynamic loads," *IEEE Trans. Smart Grid,* 2017. – the 1st paper for TSCOPF with load dynamics and uncertainty.

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Robust TSC-OPF with uncertain Dynamic Loads [7]

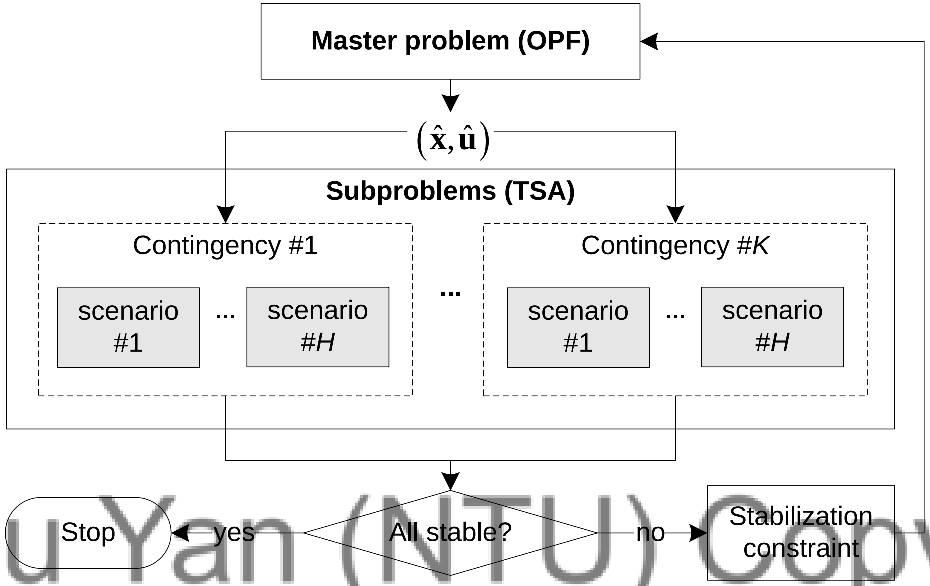


TABLE IV MEAN VALUE (μ) OF THE LOAD MODEL PARAMETERS

p_L	p_S	p_D	p_T	p_C	K_p	R_s	X_s
20%	20%	10%	10%	20%	2	0	0

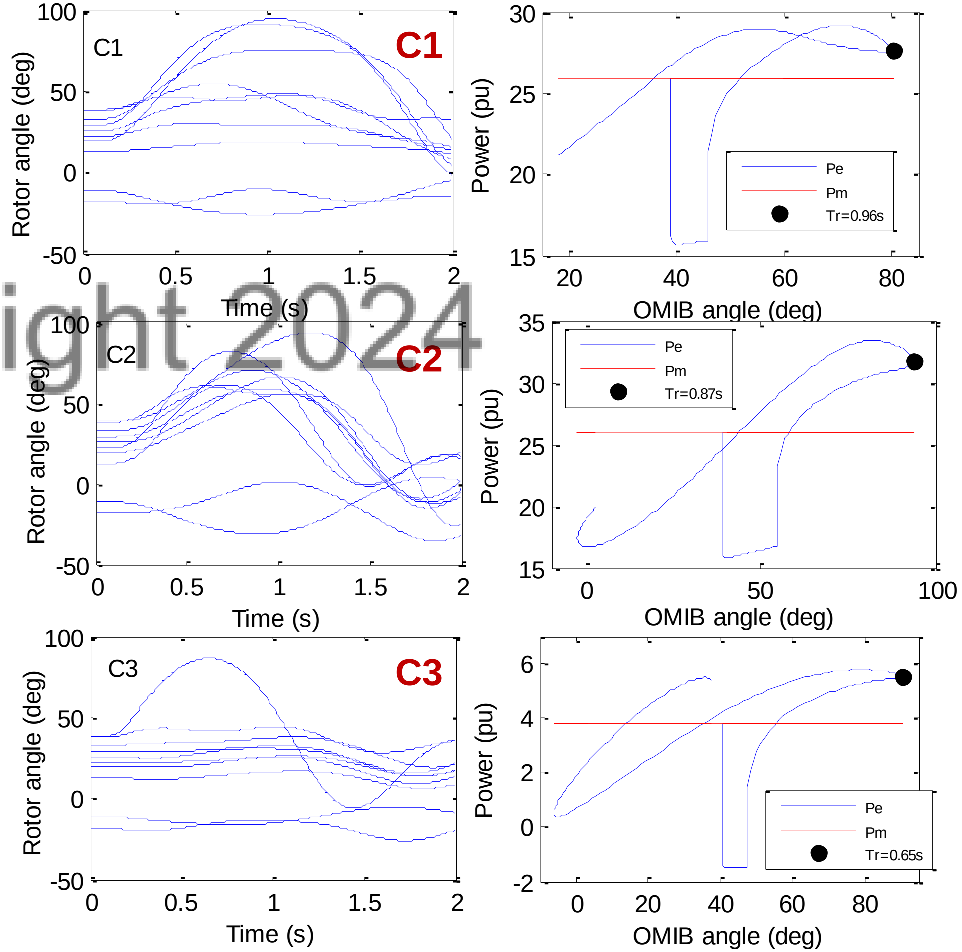
Robustness degree: $\gamma = \frac{F_i}{F} \times 100\%$

F is the total number of validation scenarios and F_i is the number of feasible (stable) scenarios for *i*-th contingency.

TABLE VII ROBUSTNESS DEGREE

Solution	C1	C2	C3	Average
This paper	99%	97%	100%	98.7%
Ref. [13]	0%	26%	22%	16%

Contingency ID	Fault location	Fault duration	Tripped line
C1	Bus 21	0.16s	Line 21-22
C2	Bus 4	0.25s	Line 4-5
C3	Bus 29	0.10s	Line 29-26



solution results: all stable

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Robust TSC-OPF with uncertain Wind Power [8]

• Augmented TSC-OPF modelling:

$$\min_{u} \ f(x, y, u, \tilde{w})$$ ← uncertain wind power output

$$\text{s.t.} \quad \mathbf{G}^{t_0}\left(y^{t_0}, u, \tilde{w}\right) = 0, \quad \mathbf{I}^{t_0}\left(x^{t_0}\right) = 0$$

$$\mathbf{H}_s^{t_0}\left(y^{t_0}, u, \tilde{w}\right) \le 0$$

$$\dot{x}(t) - \mathbf{D}\left(x(t), y(t), u, \tilde{w}\right) = 0, \quad t \in T$$

$$\mathbf{G}\left(x(t), y(t), u, \tilde{w}\right) = 0, \quad t \in T$$

$$x^{t_0} = x(t_0), \ y^{t_0} = y(t_0)$$

$$\mathbf{H}_d\left(x(t), y(t), u, \tilde{w}\right) \le 0, \quad t \in T$$

where u denotes the vector of control variables including the active output and voltage set-point of synchronous generators; $x(t)$ and $y(t)$ respectively denote the vectors of power system state and algebraic variables during the whole transient period T, where $T \in [t_0, t_{cl}) \cup [t_{cl}, t_{end}]$, t_{cl} is the fault clearance time and t_{end} is the end time of the transient period of interest, i.e., 10s; t_0 stands for the initial pre-fault steady state; $\tilde{w}$ denotes the vector of uncertain wind active power generation output. Note that the reactive power injection of the wind generators is not considered since it can be partially controlled through inverters and not an uncertain variable.

• Proposed approach:

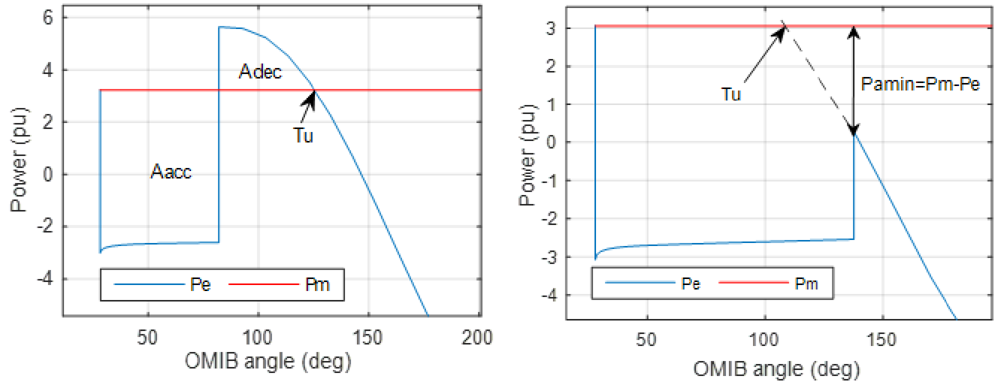
1) Robust design based on Taguchi's Orthogonal Array Testing (TOAT) for wind power uncertainty modeling (same as [7] for efficiency and effectiveness).
2) Converting the DAE set-based stability constraints to a single algebraic constraint derived from OMIB equivalent.

$$\mathbf{H}_d\left(x(t), y(t), u, \tilde{w}\right) \equiv \hat{\delta}(t_u) - \delta_{CT}(t_u) \le 0$$

$\hat{\delta}(t_u)$: targeted equivalent OMIB; $\delta_{CT}(t_u)$: reference critical OMIB (C-OMIB) under CCT

only imposed at a single time step t_u

3) Considering "extremely unstable" condition



(a) normally unstable condition; (b) extremely unstable condition

4) Decomposition-based solution framework (similar to [7]).

[8] Y. Xu, M. Yin, Z.Y. Dong, R. Zhang, and D.J. Hill, "Robust dispatch of high wind power-penetrated power systems against transient instability," *IEEE Trans. Power Syst.*, 2018. – the 1st paper for TSCOPF with wind power uncertainty.

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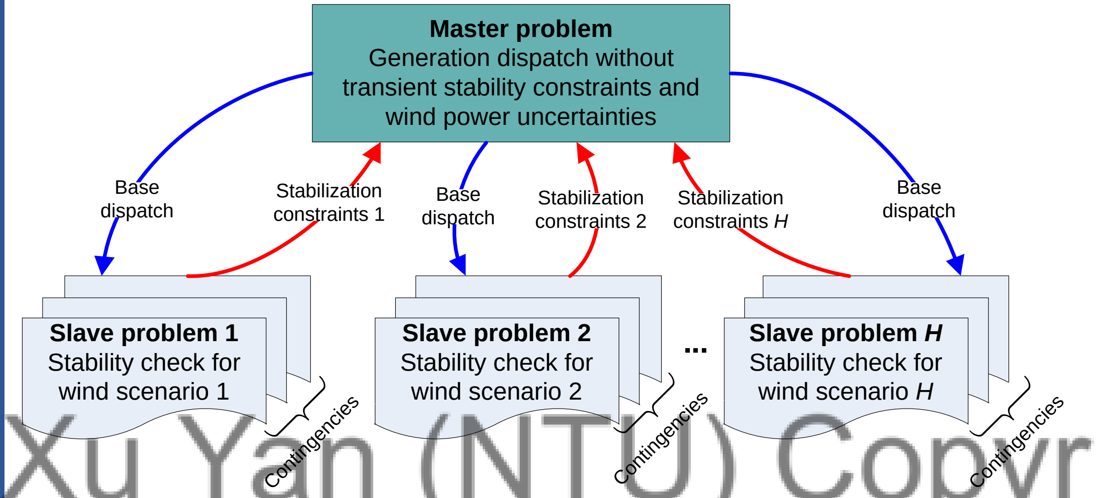
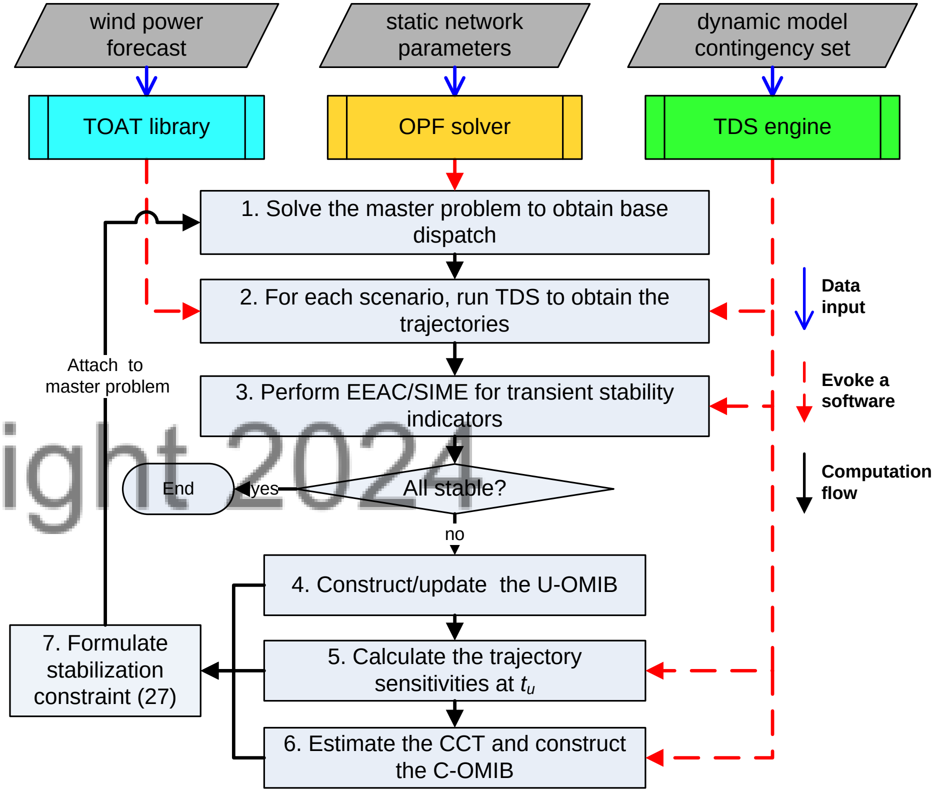
Robust TSC-OPF with uncertain Wind Power [8]

Decomposition Framework



Computation Flowchart



Computation Efficiency Analysis

$$T_{total} = (T_{OPF} + T_{TSA} \times K \times H) \times N_{It} + \sum_{i=1}^{N_{It}} (T_{TSA} \times N_{ui})$$

where T_{OPF}, T_{TSA} respectively denote the CPU time for OPF solution, and TSA for a contingency; K and H respectively denote the number of contingencies and the testing scenarios determined by TOAT; N_{It} denotes the total iteration number, and N_{ui} denotes the number of unstable scenarios in the i-th iteration. The second term means that an unstable scenario requires an additional TDS to calculate the stability margin sensitivity for deriving the stabilization constraint.

TABLE VI CPU TIME (S) FOR COMPUTATIONAL TASKS

Test system	OPF	EEAC	TDS	Total
New England	0.12	0.2	0.16	3.2
Nordic system	0.26	0.2	0.22	9.7

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Robust TSC-OPF with uncertain Wind Power [8]

- **Single Stability Constraint:**

$$\hat{\delta}(t_u)=\delta_{UT}(t_u)-\Delta\delta(t_u)\le\delta_{CT}(t_u)$$

where $\delta_{UT}(t_u)$ is the unstable OMIB angle of the original system at t_u, and $\Delta\delta(t_u)$ denotes the required reduction in the OMIB angle to be smaller than $\delta_{CT}(t_u)$.

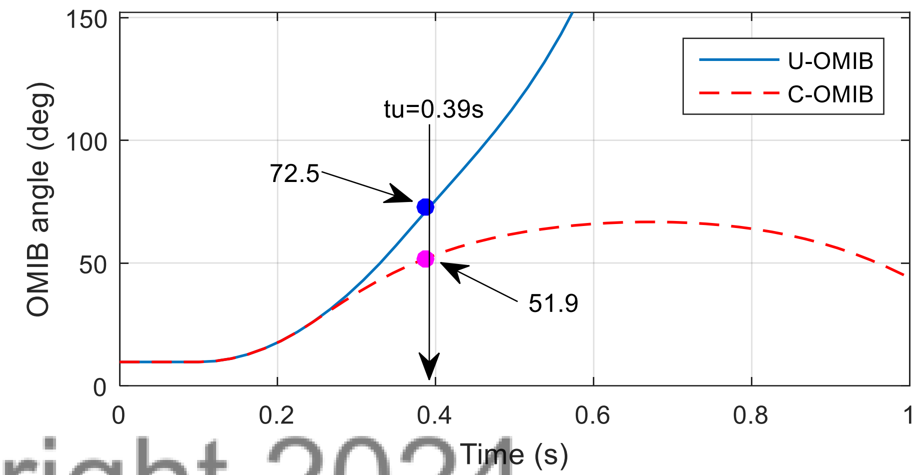
$$\Delta\delta(t_u)=\sum_{i\in S^+}\Phi_i(\delta_{UT},t_u,P_{Gi})\Delta P_{Gi} + \sum_{j\in S^-}\Phi_j(\delta_{UT},t_u,P_{Gj})\Delta P_{Gj}\ge\delta_{UT}(t_u)-\delta_{CT}(t_u)$$

where ΔP_G is the active power output change for synchronous generators, S^+ and S^- denote the sets of generators with positive and negative sensitivities, respectively (positive sensitivity means that increment in generation output will raise the OMIB angle and vice versa), $\Phi_i(\delta_{UT},t_u,P_{Gi})$ is the OMIB angle trajectory sensitivity with respect to real power output of generator i at the instability time t_u.

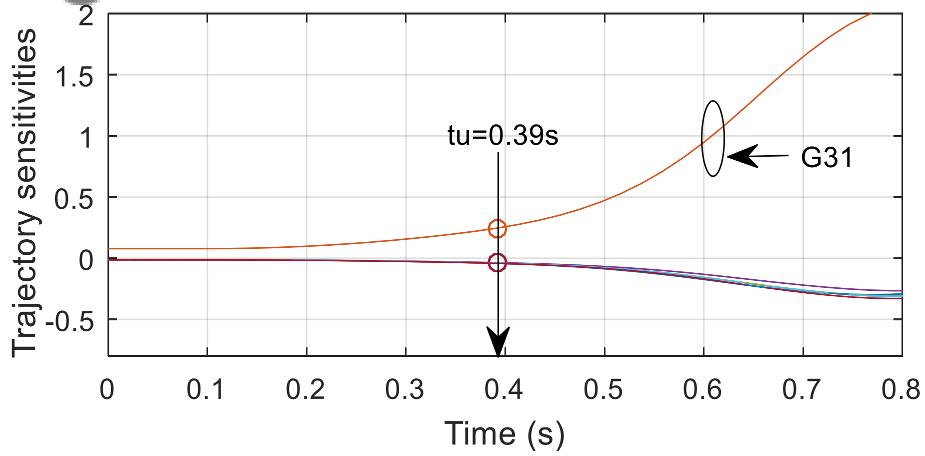
$$\Delta x(t)\approx\frac{\partial\phi(x_0,t,\lambda)}{\partial\lambda}\Delta\lambda=\Phi(x_0,t,\lambda)\Delta\lambda$$

where $\Phi(x_0,t,\lambda)$ is called as the *trajectory sensitivities* associated with the flow x.

Simulation Results on New England 39-bus System



OMIB angle trajectories for C1 under l_1



Trajectory sensitivities of synchronous machine's output to OMIB angle

[8] Y. Xu, M. Yin, Z.Y. Dong, R. Zhang, and D.J. Hill, "Robust dispatch of high wind power-penetrated power systems against transient instability," *IEEE Trans. Power Syst.*, 2018. – the 1st paper for TSCOPF with wind power uncertainty.

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Robust TSC-OPF with uncertain Wind Power [8]

Simulation Results on New England 39-bus System

TABLE I TESTING SCENARIOS DETERMINED OF OA $L_4(2^3)$

Testing scenarios	Variable levels		
	$\tilde{w}_1$	$\tilde{w}_2$	$\tilde{w}_3$
l_1	1	1	1
l_2	1	2	2
l_3	2	1	2
l_4	2	2	1

Contingency ID	Fault bus	Fault clearance	Tripped line
C1	Bus 4	0.25s	Line 4-5
C2	Bus 21	0.16s	Line 21-22
C3	Bus 29	0.10s	Line 29-26

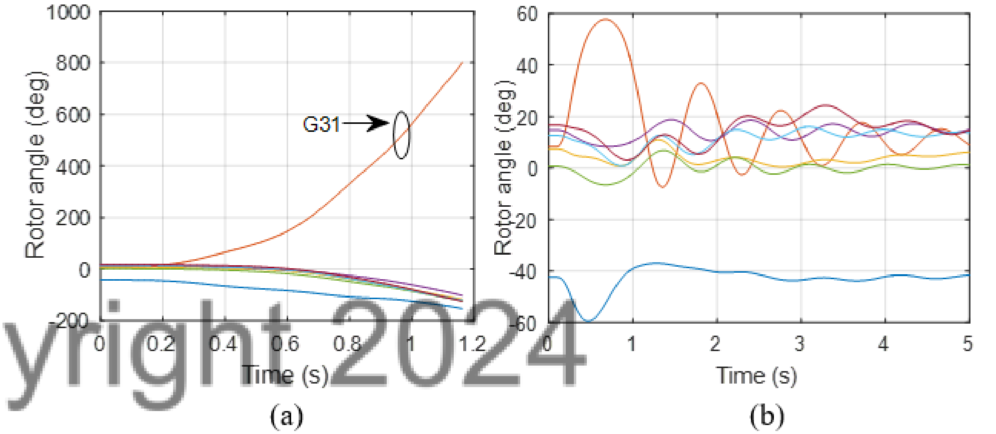

Fig.4 Rotor angle trajectories for C1 under l_1: (a)-unstable (under initial clearing time); (b)-table (under CCT)

Quasi-linearity between stability margin and key parameters

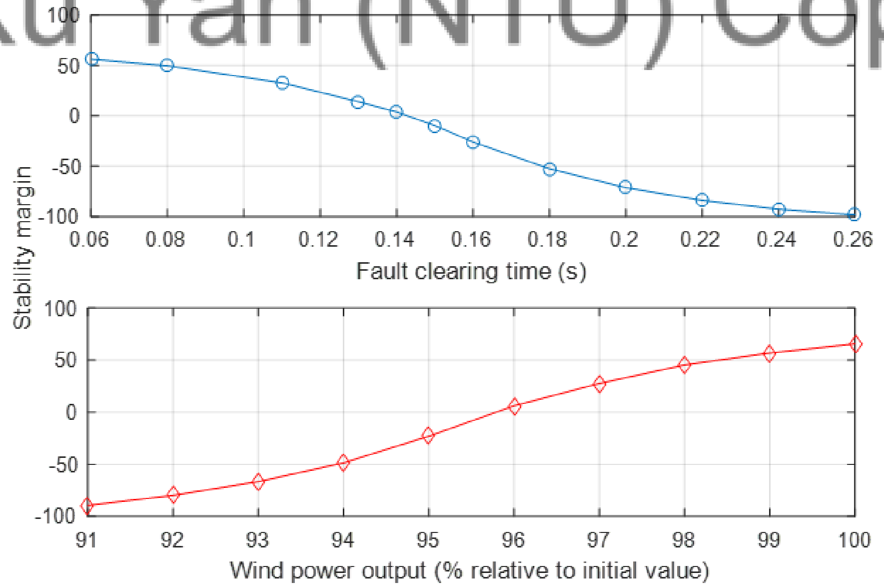

Fig.8 Transient stability margin V.S. key parameters: (a)-fault clearing time; (b)-wind power output

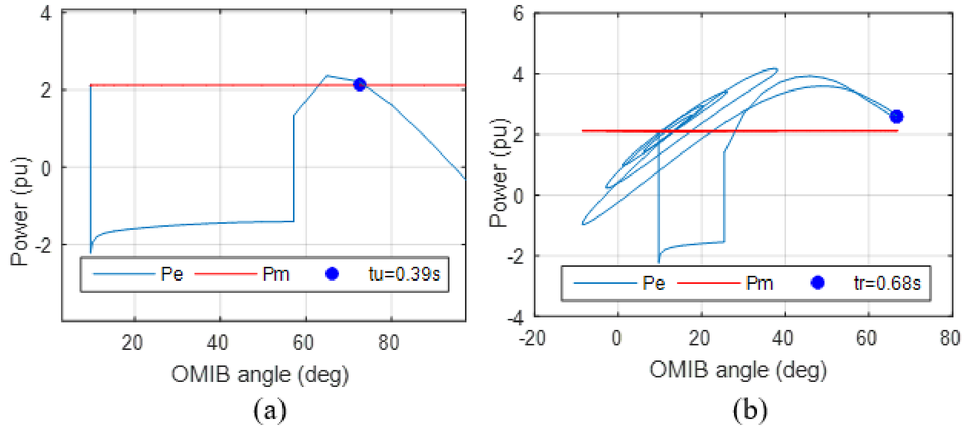

Fig.5 Pe-OMIB angle plane for C1 under l_1: (a)-unstable (under initial clearing time); (b)-stable (under CCT)

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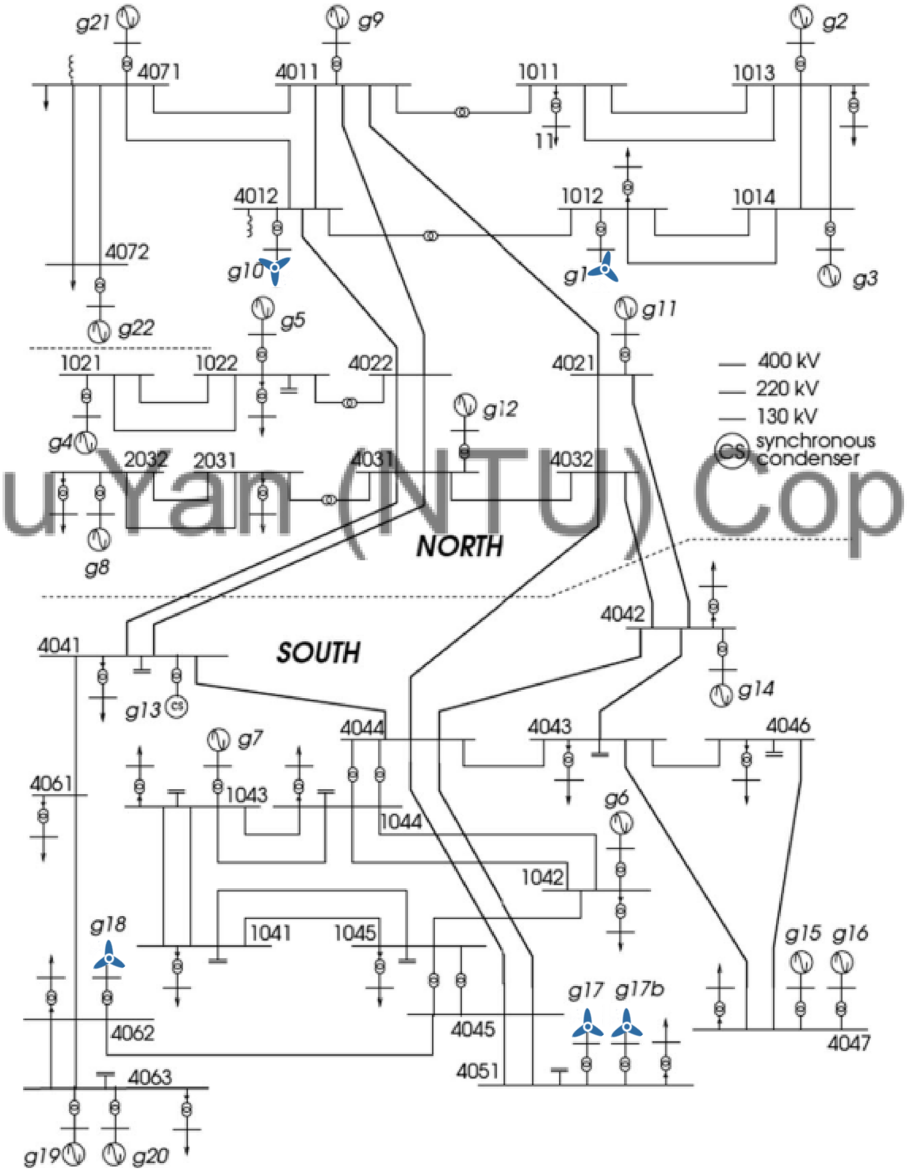
Robust TSC-OPF with uncertain Wind Power [8]

Simulation Results on Nordic32 system



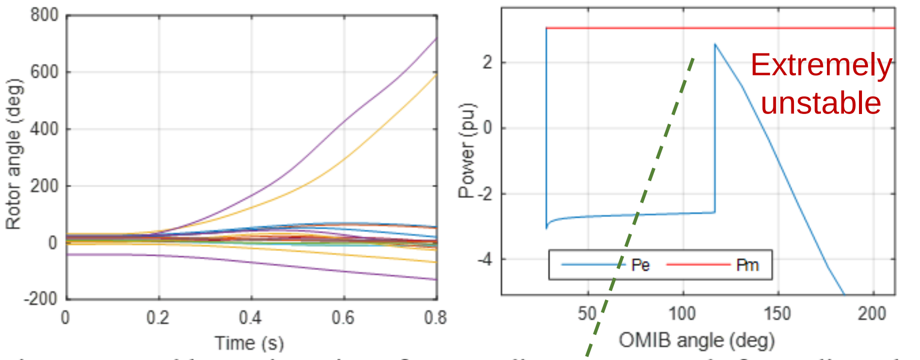


Fig.12 Unstable trajectories for Nordic32 system before dispatch: (a)-multi-machine angle; (b)-Pe-OMIB angle

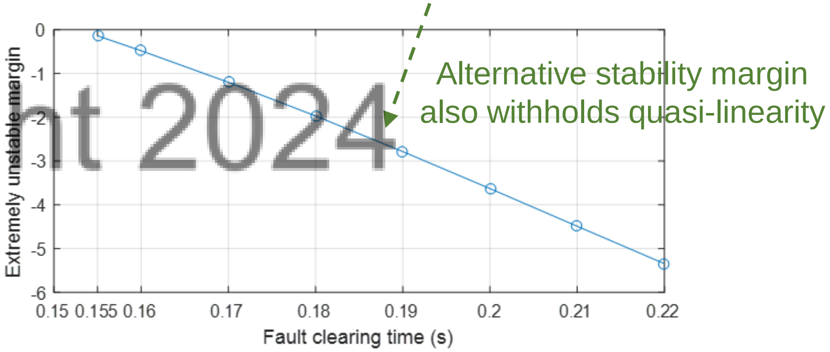


Fig.13 extremely unstable margin v.s. fault clearing time

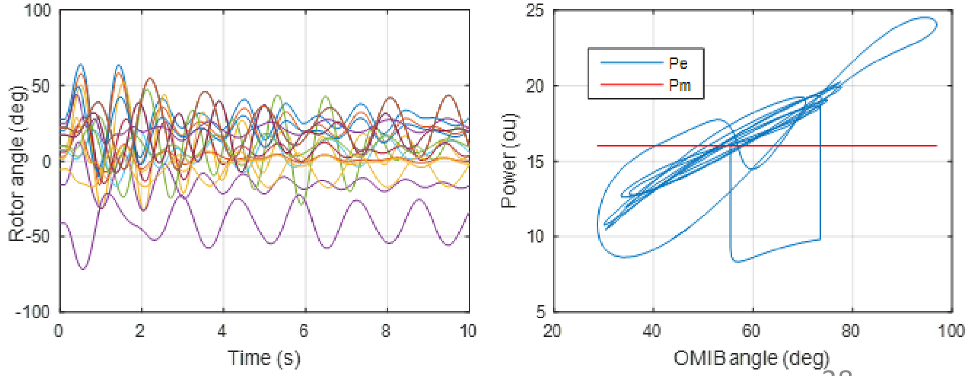


Fig.14 Illustration of stable trajectories for Nordic32 system after dispatch: (a)-multi-machine angle; (b)-Pe-OMIB angle

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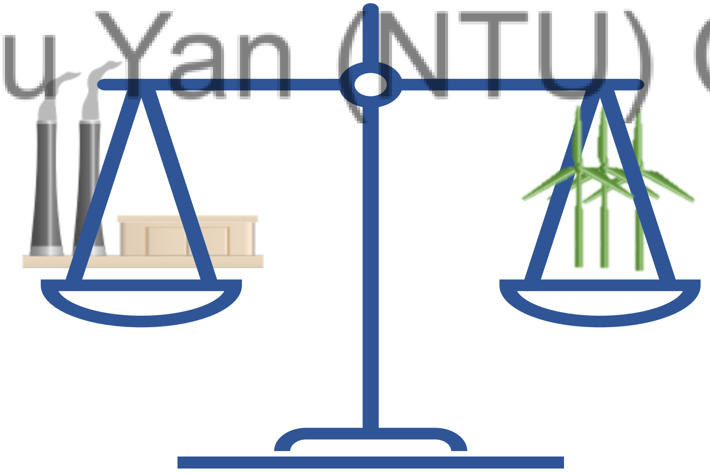
Preventive Transient Stability Control against Wind Power Variation [9]

- **Aim: preventively redispatch the power system for compensating wind power variation**

$$\Delta P = -\sum_{i=1}^{n} \Delta P_{gi} = \sum_{j=1}^{K} \Delta P_{wj}$$

The total change of the wind power will be compensated by synchronous generators



possible strategies

1. By all synchronous machines according to their capacity ratio:

- $\Delta P_{gi} = -\frac{\bar{P}_{gi}}{\sum_{i=1}^{n} \bar{P}_{gi}} \cdot \Delta P$

2. By all synchronous machines according to their inertia:

- $\Delta P_{gi} = -\frac{M_i}{\sum_{i=1}^{n} M_i} \cdot \Delta P$

3. By all synchronous machines evenly:

- $\Delta P_{gi} = -\frac{\Delta P}{n}$

4. By critical machines and non-critical machines:

- $$\begin{cases} \Delta P_{cgr} = -\frac{\bar{P}_{cgr}}{\sum_{r \in C} \bar{P}_{cgr}} \cdot \Delta P & \Delta P \geq 0 \\ \Delta P_{ngq} = -\frac{\bar{P}_{ngq}}{\sum_{q \in N} \bar{P}_{ngq}} \cdot \Delta P & \Delta P \leq 0 \end{cases}$$

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Trajectory Sensitivity of a Dynamic System

The dynamic behaviour of a power system can be described by the following DAEs

$$\dot{x} = f(x, y, \lambda) \tag{1}$$

$$0 = g(x, y, \lambda) \tag{2}$$

where x denotes dynamic state variables, for example, generator angles and speeds; y denotes algebraic state variables, for example, load bus voltage magnitudes and angles; and λ represents parameter changes.

The flow of x and y can be defined as follows [23]

$$x(t) = \phi(x_0, t, \lambda) \tag{3}$$

$$y(t) = \varphi(x_0, t, \lambda) \tag{4}$$

where $x(t)$ and $y(t)$ satisfy (1) and (2), along with the initial conditions

$$\phi(x_0, t_0, \lambda) = x_0 \tag{5}$$

$$g\big(\phi(x_0, t_0), \varphi(x_0, t_0); \lambda\big) = 0 \tag{6}$$

To obtain the sensitivities of the flows ϕ and φ, the Taylor series expansions of (5) and (6) are formed [23]

$$\phi(x_0, t, \lambda + \Delta\lambda) = \phi(x_0, t, \lambda) + \frac{\partial \phi(x_0, t, \lambda)}{\partial \lambda}\Delta\lambda + \varepsilon^{\phi} \tag{7}$$

$$\varphi(x_0, t, \lambda + \Delta\lambda) = \varphi(x_0, t, \lambda) + \frac{\partial \varphi(x_0, t, \lambda)}{\partial \lambda}\Delta\lambda + \varepsilon^{\varphi} \tag{8}$$

where ε^{ϕ} and ε^{φ} are the higher-order terms of the Taylor series expansion.

For small $\|\Delta\lambda\|$, the higher-order terms ε^{ϕ} and ε^{φ} can be neglected without much sacrifice of accuracy, giving

$$\begin{aligned}\Delta x(t) &= \phi(x_0, t, \lambda + \Delta\lambda) - \phi(x_0, t, \lambda) \\ &\simeq \frac{\partial \phi(x_0, t, \lambda)}{\partial \lambda}\Delta\lambda \equiv \Phi(x_0, t, \lambda)\Delta\lambda \end{aligned} \tag{9}$$

$$\begin{aligned}\Delta y(t) &= \varphi(x_0, t, \lambda + \Delta\lambda) - \varphi(x_0, t, \lambda) \\ &\simeq \frac{\partial \varphi(x_0, t, \lambda)}{\partial \lambda}\Delta\lambda \equiv \Psi(x_0, t, \lambda)\Delta\lambda \end{aligned} \tag{10}$$

where the time-varying partial derivatives Φ and Ψ are called trajectory sensitivities associated with the flows x and y [23].

[23] I. A. Hiskens and M. A. Pai, "Trajectory sensitivity analysis of hybrid systems," *IEEE Trans. Circuits and Systems I: Fundamental Theory and Applications*, 2000.

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Trajectory Sensitivity of New England System with Wind Power Variation

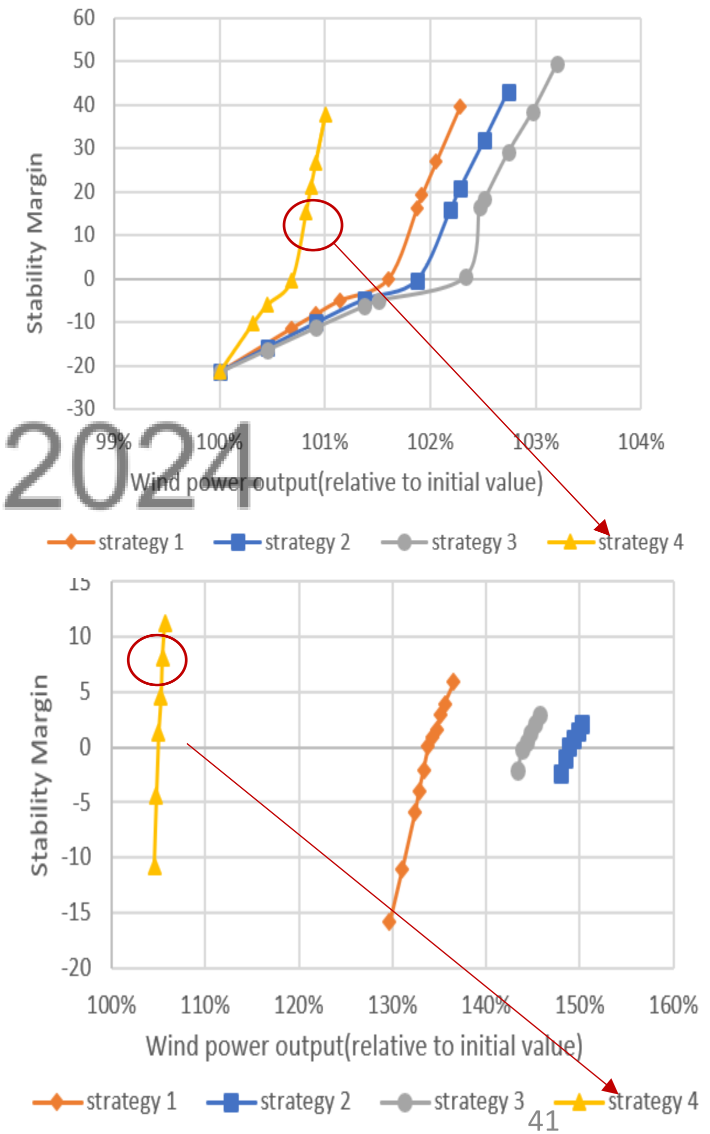
Contingency	Fault bus	Clearance	Tripped line
C1	Bus 21	0.16s	Line 21-22
C2	Bus 29	0.10s	Line 29-26

$$\phi = \frac{\Delta\eta}{\Delta P}$$

Sensitivity of different strategies

Strategy	C1	C2
1	2534.7	326.1
2	2188.4	193.5
3	2042.7	202.4
4	5609.2	1894.4

Stability margin VS. wind power output



- High sensitivity represents efficient and potent wind power balancing strategy
- Strategy 4 is the most efficient → compensated by critical machines and non-critical machines

H. Yuan, Y. Xu, et al, "Sensitivity analysis of transient stability for power systems with high level wind power," *11th IET Conf. APSCOM, 2018.*

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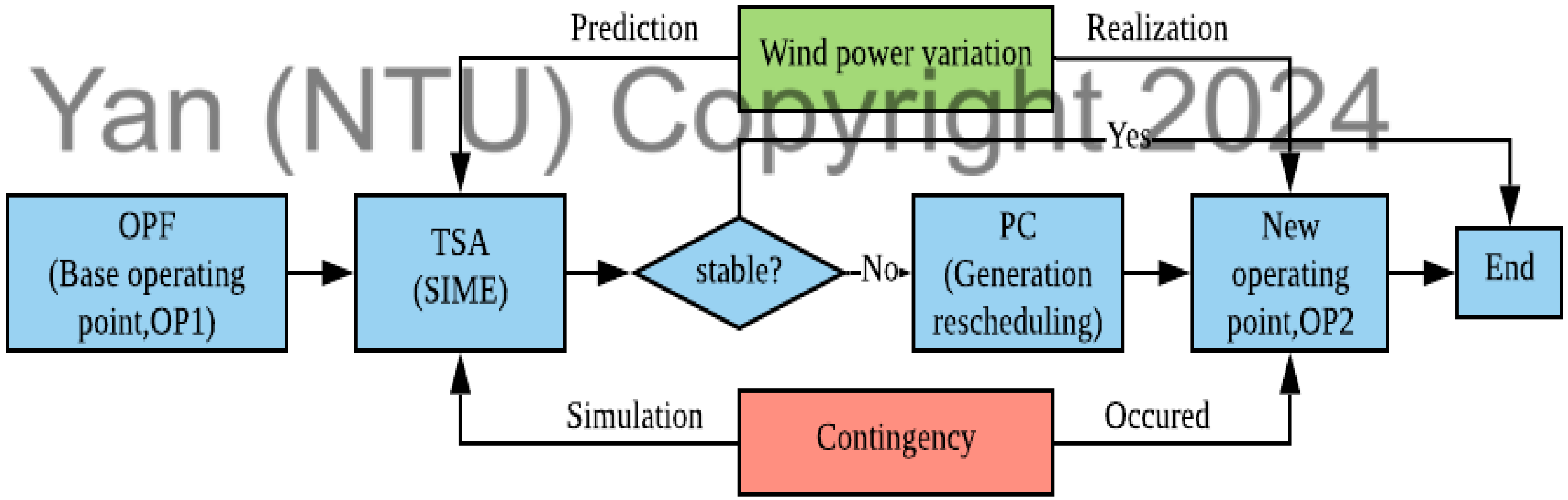
Preventive Transient Stability Control against Wind Power Variation [9]

- Based on trajectory sensitivity analysis, the wind power balancing by CM and NM is the most efficient.
- According to trajectory sensitivity, CM and NM can be recognized by its negative and positive value.
- Hence, preventive control (PC) can be applied by the recognized CM and NM.



Proposed Preventive Redispatch Framework

[9] H. Yuan, Y. Xu, "Trajectory Sensitivity based Preventive Transient Stability Control of Power Systems against Wind Power Variation," *Int. J. Electrical Power and Energy Systems,* 2020.

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Preventive Transient Stability Control against Wind Power Variation [9]

$$\min C_p = \sum_{i}^{N_G} a_k P_{G_k}^2 + b_k P_{Gk} + c_k$$

s.t. $\Delta P_{Gi} \geq 0, \quad i \in S^+$ Positive sensitivity

$\Delta P_{Gj} \leq 0, \quad j \in S^-$ Negative sensitivity

$\sum_k \Delta P_{Gk} + \Delta P_w = 0, \quad \forall k \in \{S^- \cup S^+\}$ Wind power balancing

$\Delta\eta + \eta_0 \geq 0$

$\Delta\eta = u(\Delta P_{Gk})$

Transient stability constraint

$P_{Gk}^{\min} \leq P_{Gk}^0 + \Delta P_{Gk} \leq P_{Gk}^{\max}$

$0 \leq \Delta P_{Gk} \leq 50$

Generation limit

Linearization of transient stability constraints

$$\Phi(x_0, t, \lambda) = \frac{\phi(x_0,t,\lambda+\Delta\lambda)-\phi(x_0,t,\lambda)}{\Delta\lambda}$$

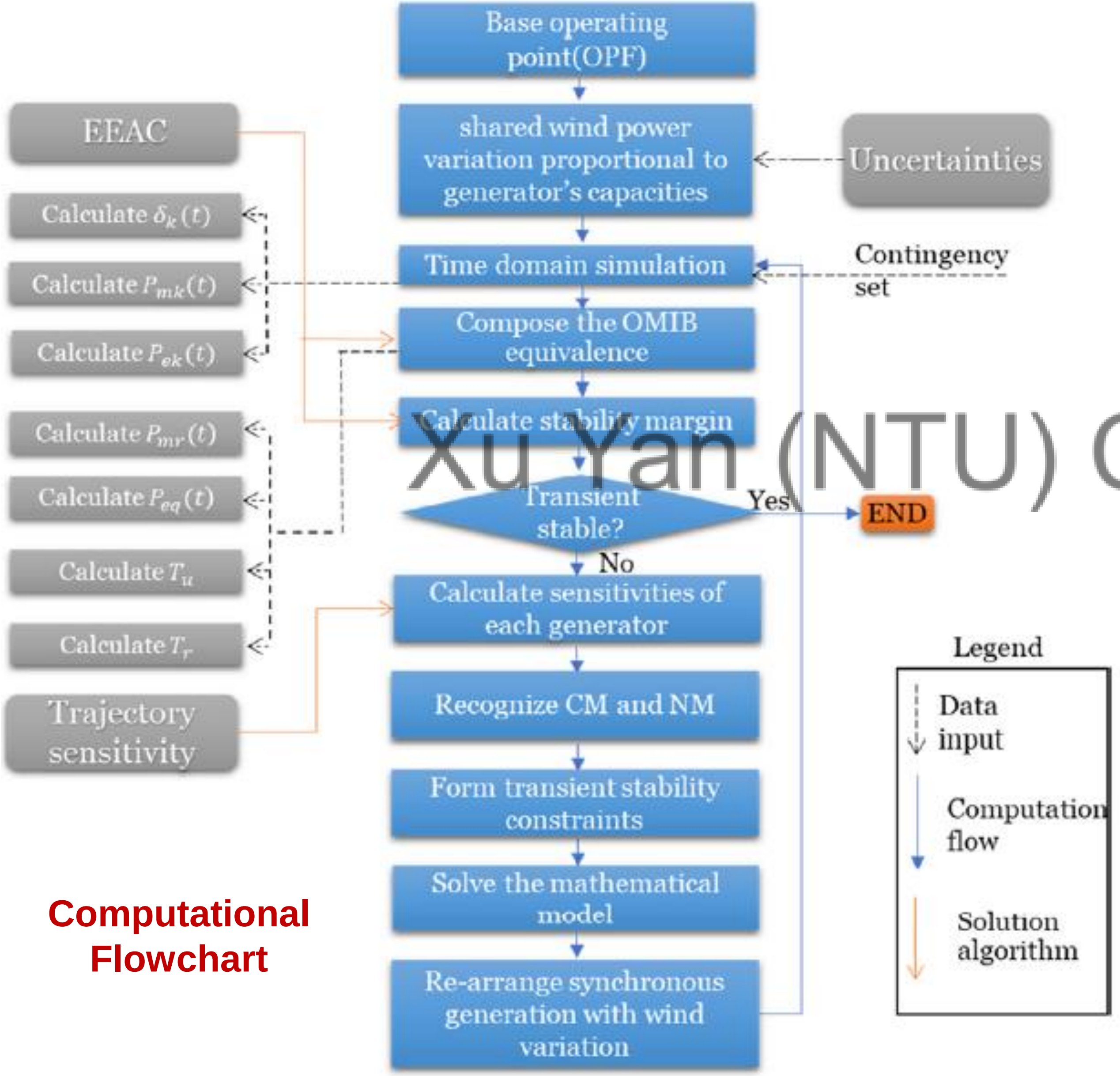
$$\Phi(\eta, \lambda) \cdot \Delta\lambda = \Delta\eta$$

$$\Delta\eta = \sum_k^{N_G} \Phi_i(\eta, P_{Gk})\, \Delta P_{Gk}$$

$$\sum_k \Phi_k(\eta, P_{Gk}) \Delta P_{Gk} + \eta_0 \geq 0$$

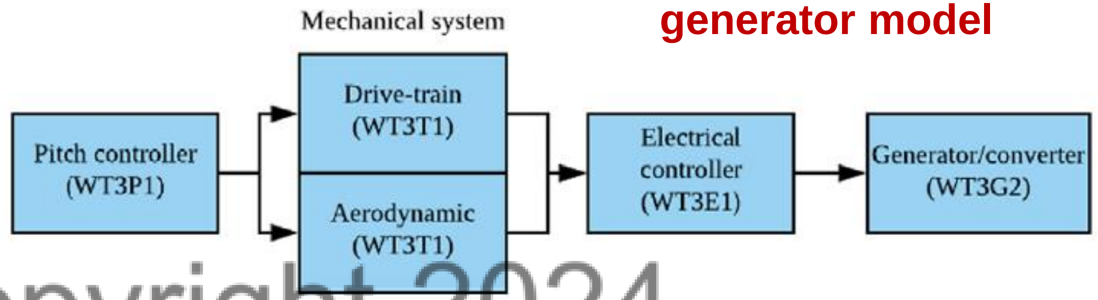
[9] H. Yuan, Y. Xu, "Trajectory Sensitivity based Preventive Transient Stability Control of Power Systems against Wind Power Variation," *Int. J. Electrical Power and Energy Systems*, 2020.

Preventive Transient Stability Control against Wind Power Variation [9]



Computational Flowchart

Wind power generator model



Contingency Set for New England 39-bus system

Contingency ID	Fault bus	Fault clearance	Tripped line
C1	Bus 4	0.38 s	Line 4–5
C2	Bus 21	0.14 s	Line 21–22
C3	Bus 29	0.05 s	Line 29–26

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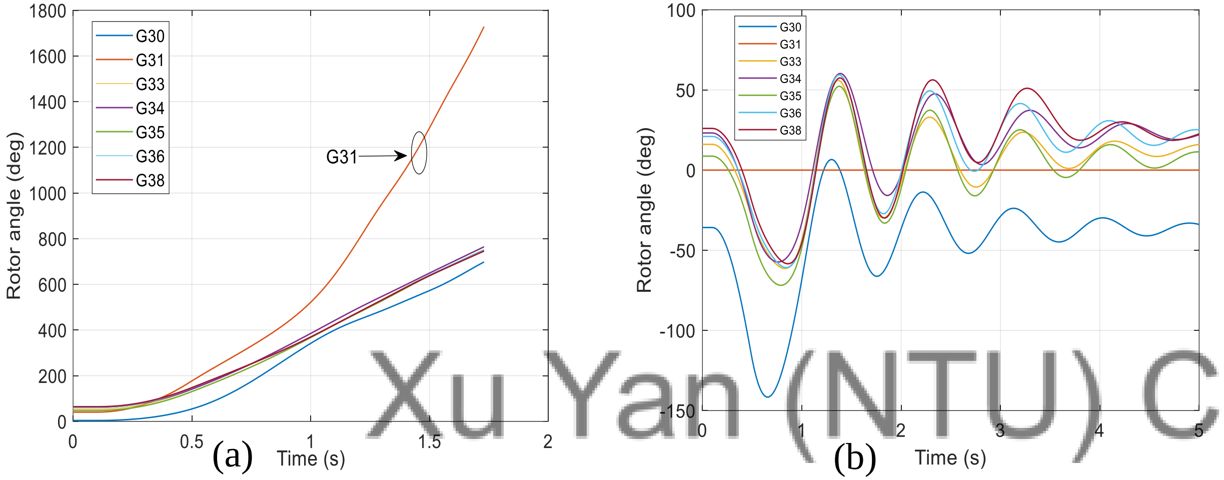
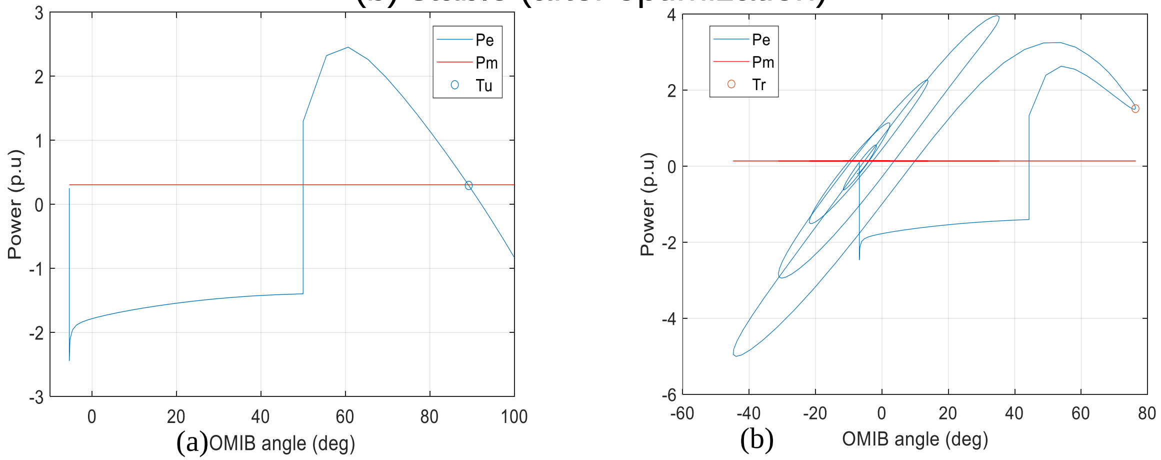
Preventive Transient Stability Control against Wind Power Variation [9]

- **Results: Single-contingency**



Rotor angle trajectories for C1: (a) unstable (before optimization); (b) stable (after optimization)



Pe-OMIB angle plane for C1: (a) unstable (before optimization); (b) stable (after optimization)

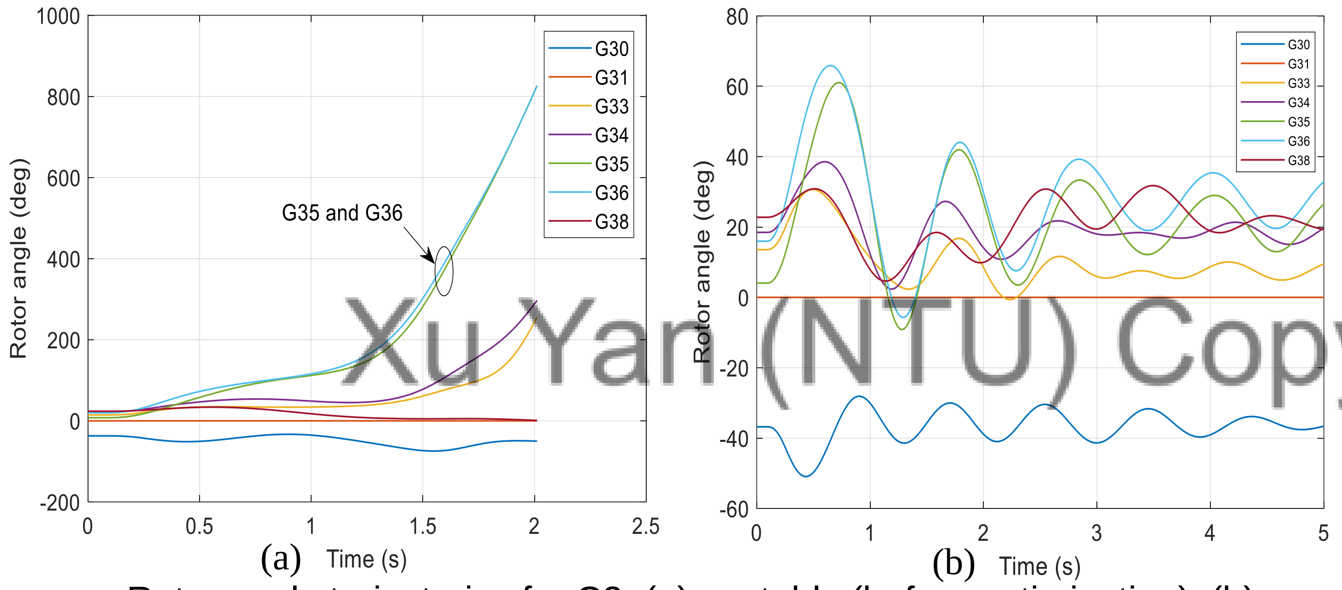
Generation output (MW) at base operating point, before and after optimization with wind variation for C1

Gens	Base	Before	After
G30	239.5	250.4	241.4
G31#	560.9	583.5	567.5
G32*	650	600	600
G33	624	647.4	654.5
G34	504.1	524.4	524.2
G35	644.9	668.4	665.9
G36	553	576.4	576.7
G37*	540	490	490
G38	822.4	850.5	872.4
G39*	1000	950	950
Wind variation	0	-150	-150
Stability margin	1.18	-22.69	12.86

- Contingency 1 is applied
- CM is G31
- Reduce G31 generation to improve transient stability

Preventive Transient Stability Control against Wind Power Variation [9]

- **Results: Single-contingency**



Rotor angle trajectories for C2: (a) unstable (before optimization); (b) stable (after optimization)

Generation output (MW) at base operating point, before and after optimization with wind variation for C2

Gens	Base	Before	After
G30	239.5	254.1	289.5
G31#	560.9	591.1	612.9
G32*	650	583.3	583.3
G33	624	655.2	674
G34	504.1	531.2	504.1
G35	644.9	676.2	644.9
G36	553	584.2	553
G37*	540	473.3	473.3
G38	822.4	859.9	872.4
G39*	1000	933.3	933.3
Wind variation	0	-200	-200
Stability margin	45.26	-37.8	48.85

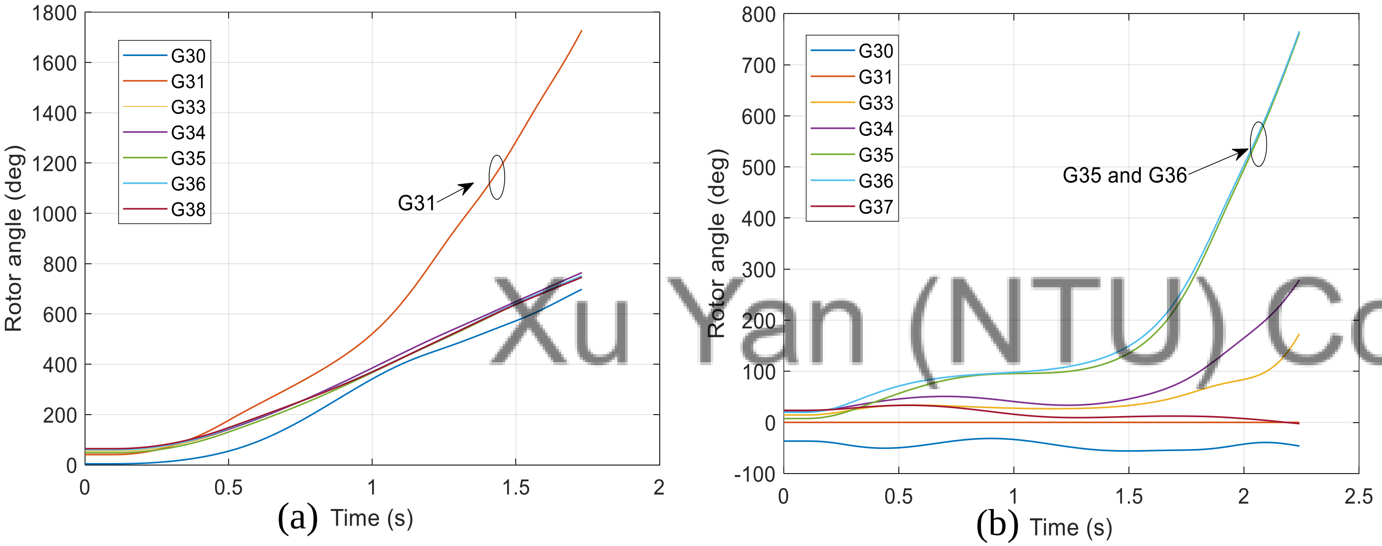
Sensitivities of seven synchronous generators for C1 and C2

Generator	G30	G31	G33	G34	G35	G36	G38
C1:Sensitivity	6.359	0	1.385	1.36	1.398	1.39	1.355
C2:Sensitivity	0.2295	0	0.0145	-0.0158	-1.947	-1.3315	0.228

- Contingency 2 is applied
- CM is G35 and G36
- Reduce G35 and G36 generation for stabilization

Preventive Transient Stability Control against Wind Power Variation [9]

- **Results: Multi-contingency**



Rotor angle trajectories for multi-contingency before optimization: (a) C1; (b) C2

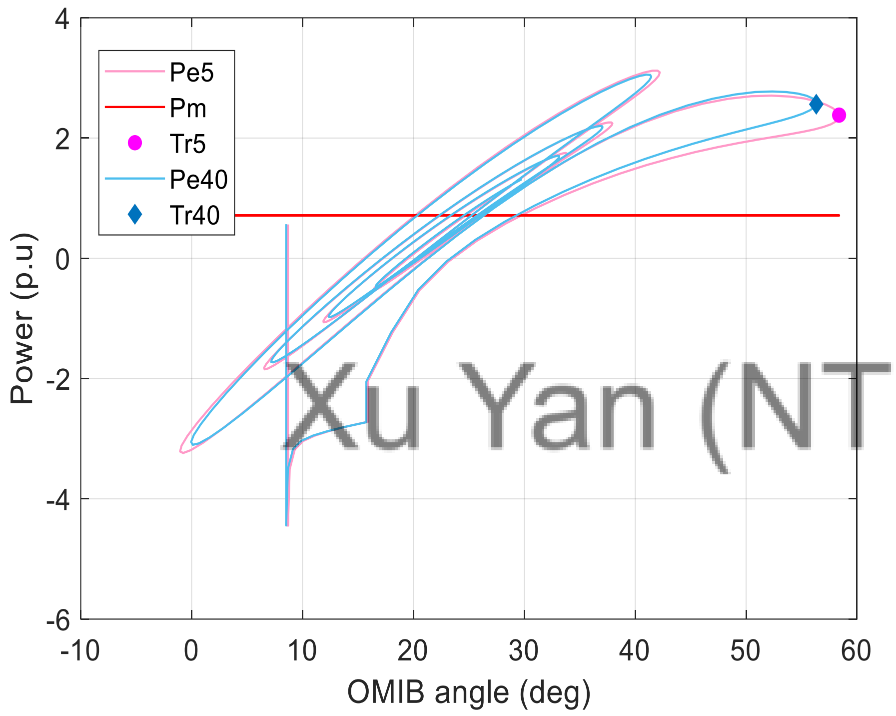
Generation output (MW) and wind variation (MW) for multi-contingency before and after optimization

Gens	Before	After
G30	250.4	298.2
G31#	583.5	480.5
G32*	600	600
G33	647.4	668.8
G34	524.44	566.85
G35	668.37	657.43
G36	576.4	569.3
G37*	490	490
G38	850.5	862.6
G39*	950	950
Wind variation	-150	-150
Stability margin	-22.7(C1) -5.71(C2) 9.07(C3)	64.19(C1) 24.38(C2) 0.01(C3)

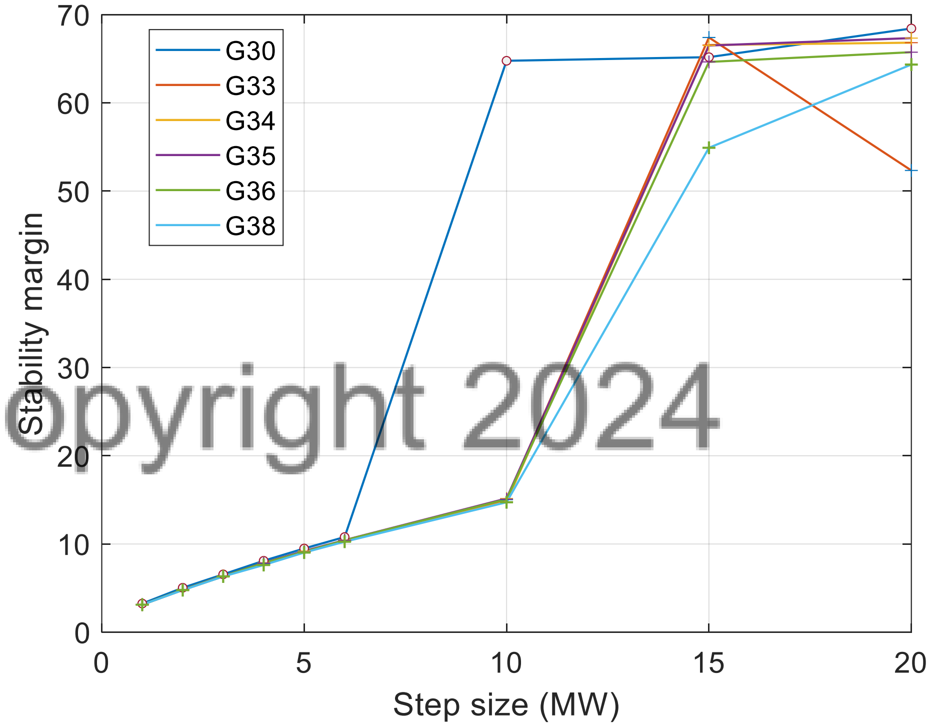
G31, G35, and G36 are the critical. Compared to the dispatch of each generator before and after the optimization, it can be found that G31, G35, and G36 decrease their generations, 103MW, 10.9MW, and 7.1MW, separately, verifying that decrease the generation of critical generators can improve the stability.

Preventive Transient Stability Control against Wind Power Variation [9]

❖ Results: control accuracy



Pe-OMIB angle plane for C2 with the step size of 5MW and 40MW



The stability margin VS step size for contingency 1

- For C2, the stability margin after redispatch is 48.85, which is too large. To achieve a less conservative result, the step size is reduced to 5MW and re-perform the proposed approach. the final stability margin is reduced to **36.78**.

- The stability margin varies linearly against step size within a small range, which is from 1MW to 6MW. Hence, in this case, the step size can be chosen as 5MW to obtain a reasonable stability margin after the optimization.

Preventive and corrective coordinated transient stability dispatch against uncertain wind power [10]

- **Motivation: coordinating PC and CC for transient stability under wind power uncertainty**

Preventive Control and Corrective Control

Corrective control (CC)

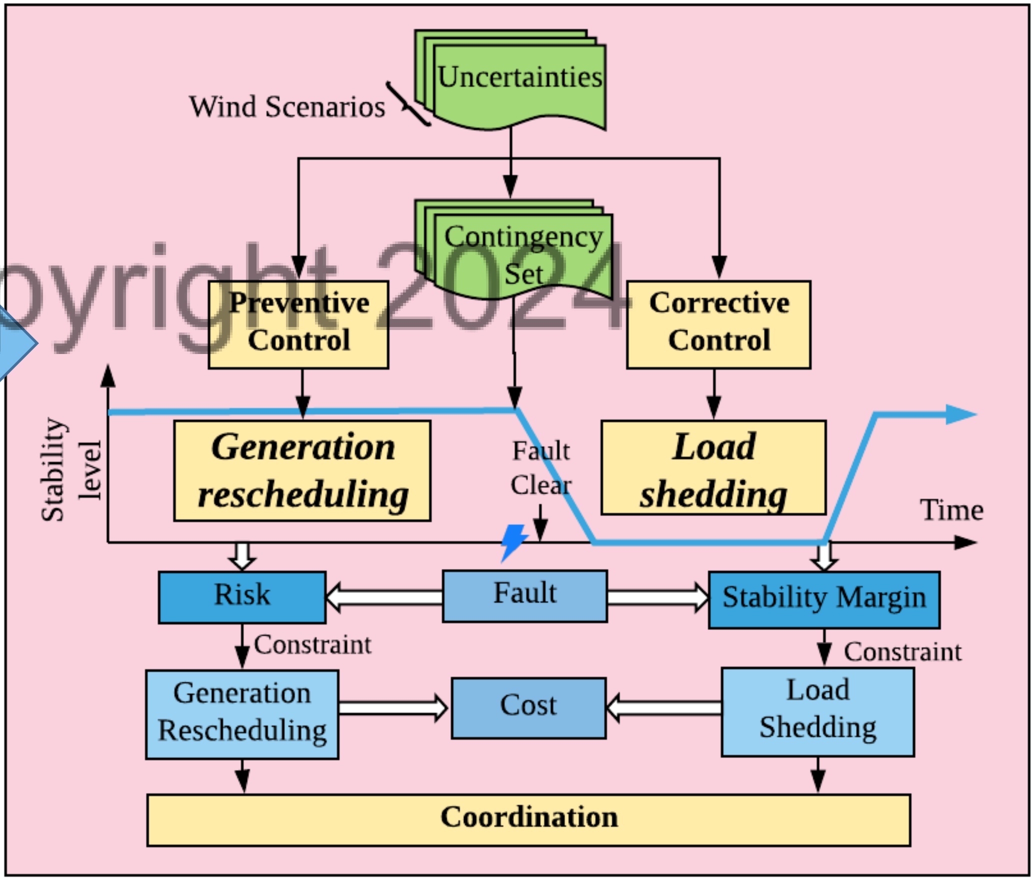
short-term cost is very high, but long-term cost is low since the probability of fault is low.

Cost

Preventive control (PC)

Short-term cost is low but long-term cost is high since it is continuously implemented.

Coordination of PC and CC



Proposed framework for coordination of PC and CC

[10] H. Yuan, Y. Xu, "Preventive-Corrective Coordinated Transient Stability Dispatch of Power Systems with Uncertain Wind Power," *IEEE Trans. Power Syst.*, 2020.

Preventive and corrective coordinated transient stability dispatch against uncertain wind power [10]

- **Mathematical model: Bi-level two-step optimization**

Upper level

$$\min_{\tau} \quad C = C_p + C_c$$
$$s.t. \quad -1 \le \tau \le 0$$

Coordination variable

Compact model

$$\min_{x_u \in X_U, x_l \in X_L} F(x_u, x_l, \tilde{w})$$
$$s.t. G_m = (x_u, x_l, \tilde{w}) \le 0,$$
$$m = 1, \dots, M \quad \textit{upper level}$$
$$x_l \in \underset{x_l \in X_L}{\operatorname{argmin}} \{ f_1(x_u, x_l, \tilde{w}), \ f_2(x_u, x_l, \tilde{w}):$$
$$g_n(x_u, x_l, \tilde{w}) \le 0, n = 1, \dots, N\}$$
lower level

Lower level

Preventive control (First step)

$$\min_{\Delta P_{Gi}} \quad C_p = \sum_{i}^{N_G} a_i P_{G_i}^2 + b_i P_{Gi} + c_i$$
$$s.t. \quad P_{Gi} = P_{Gi}^0 + \Delta P_{Gi}$$
$$P_{Gi} + \tilde{P}_{Wi} - P_{Di} = V_i \sum_{j=1}^{N_B} V_j (G_{ij} cos\theta_{ij} + B_{ij} sin\theta_{ij})$$
$$Q_{Gi} + \tilde{Q}_{Wi} - Q_{Di} = V_i \sum_{j=1}^{N_B} V_j (G_{ij} cos\theta_{ij} - B_{ij} sin\theta_{ij})$$
$$\sum_{i}^{N_G} \Delta P_{Gi} = 0$$
$$P_{Gi}^{min} \le P_{Gi} \le P_{Gi}^{max}, \quad i = 1,2, \dots, N_G$$
$$\Delta R + \tau R_0 \le 0$$
Risk constraint
$$\Delta R = \sum_{k=1}^{Nc} -\Delta\eta_k * p_k$$
$$\Delta\eta_k = u(\Delta P_{Gi})$$

Corrective control (Second step)

$$for\ k, \quad \min_{\Delta P_{Di}} C_c = \sum_{j=1}^{H} \cdot \sum_{i=1}^{N_D} p \times c_D \Delta P_{Di}(l_j)$$
$$P_{ei}^t + \tilde{P}_{Wi} - P_{Di}^t = V_i^t \sum_{j=1}^{N_B} V_j^t (G_{ij} cos\theta_{ij}^t + B_{ij} sin\theta_{ij}^t)$$
$$Q_{ei}^t + \tilde{Q}_{Wi} - Q_{Di}^t = V_i^t \sum_{j=1}^{N_B} V_j^t (G_{ij} cos\theta_{ij}^t - B_{ij} sin\theta_{ij}^t)$$
$$P_{Di} \ge \Delta P_{Di} \ge 0$$
$$\Delta\eta + \eta_0 \ge 0$$
$$\Delta\eta = v(\Delta P_{Di})$$
Stability constraint

Risk index

$$R = \sum_{k=1}^{N_c} p_k \times (-\eta_k)$$

Uncertainty modelling

$$\eta_k = z(\eta_{k0})$$

Preventive and corrective coordinated transient stability dispatch against uncertain wind power [10]

- **Solution algorithm**

Linearization of risk and stability constraints

$$\Delta\lambda = \frac{\Delta\eta}{\Phi(\eta,\lambda)}$$

$$\Delta\eta_k = \sum_{i}^{N_G} \Phi_i(\eta, P_{Gi})\, \Delta P_{Gi}$$

$$\Delta R = \sum_{k=1}^{Nc} -\Delta\eta_k * p_k$$

$$\Delta R \le \tau R_0$$

$$\sum_{k=1}^{Nc} \cdot \left(-\sum_{i}^{N_G} \Phi_i(\eta, P_{Gi})\, \Delta P_{Gi} \right)_k * p_k \le \tau R_0$$

Risk constraint

$$\Delta\eta = \sum_{i}^{N_D} \Phi_i(\eta, P_{Di})\, \Delta P_{Di}$$

$$\Delta\eta + \eta_0 \ge 0$$

$$\sum_{i}^{N_D} \Phi_i(\eta, P_{Di}) \Delta P_{Di} + \eta_0 \ge 0$$

Stability constraint

Mathematical model solving-golden section search:

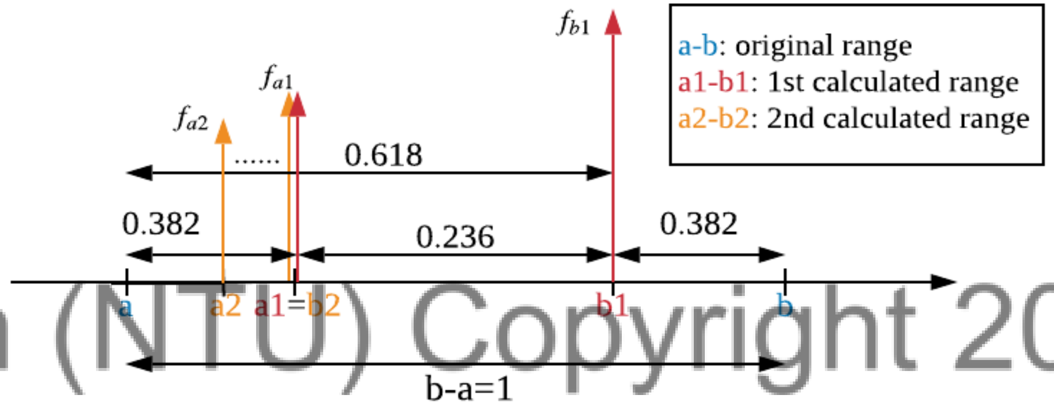


Illustration of the golden section search

Golden section search is to search risk coordination variable τ
→ simple and efficient for unimodal optimization

Uncertainty modelling-TOAT algorithm:

Testing scenarios determined of OA L_4(2^3)

Testing scenarios	Variable levels		
	$\tilde{\mathbf{w}}_1$	$\tilde{\mathbf{w}}_2$	$\tilde{\mathbf{w}}_3$
l_1	1	1	1
l_2	1	2	2
l_3	2	1	2
l_4	2	2	1

- TOAT is to model the uncertainty of wind power
- Representative scenarios are modelled
- Transient stability margin under all scenarios can be calculated:

$$\eta_k = \frac{1}{H} \sum_{i=1}^{H} \eta_k(l_i)$$

Preventive and corrective coordinated transient stability dispatch against uncertain wind power [10]

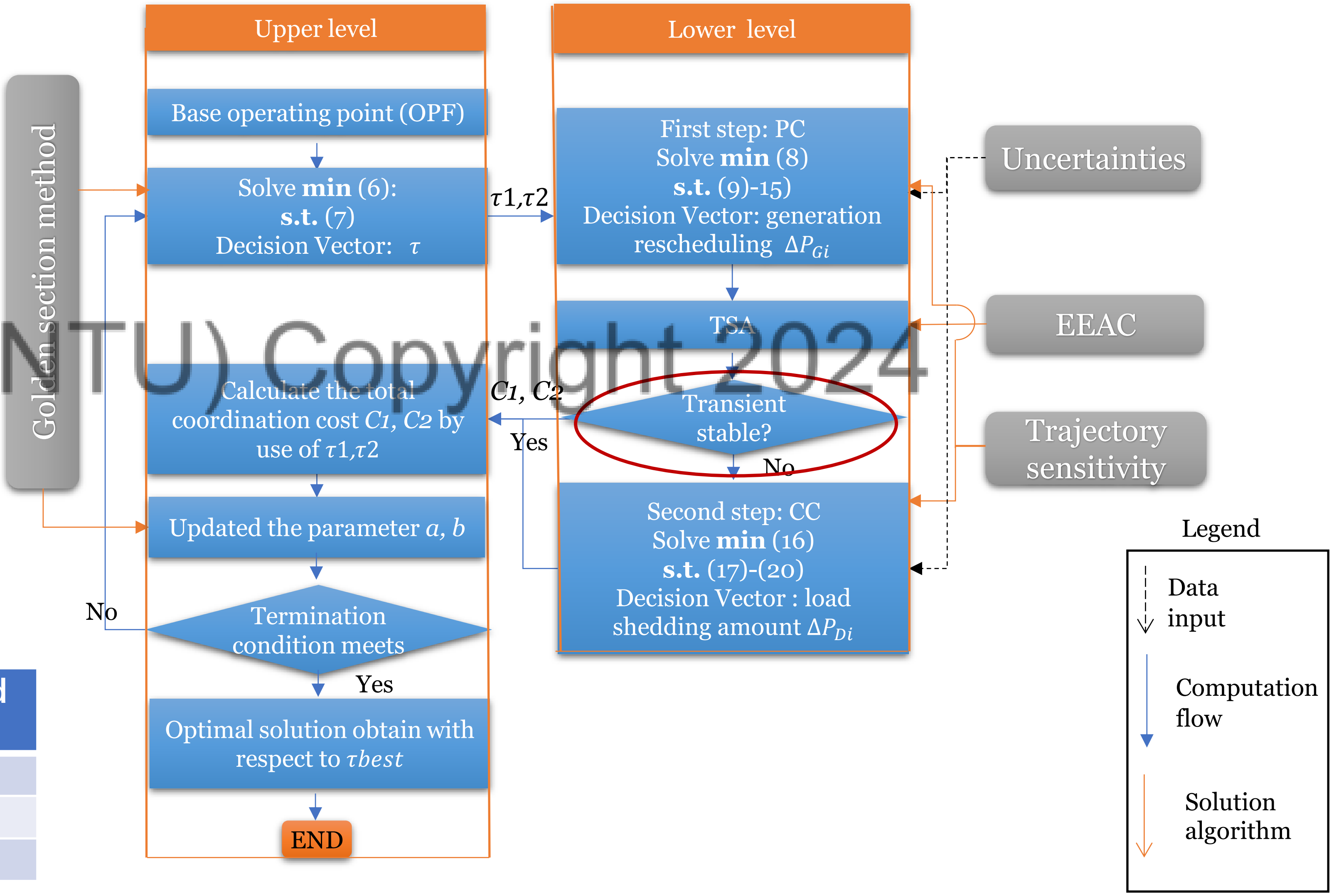
- **Computation process**

Remarks:

- CC may be not needed as the PC alone is adequate to stabilize the system. In this case the cost of CC is zero
- C1 and C2 are the total coordination cost for PC and CC with respect to $\tau1$ and $\tau2$

Contingency set

Contingency ID	Fault bus	Fault clearance	Tripped line
#1	29	0.1	29-26
#2	28	0.1	28-26
#3	28	0.1	28-29



Flowchart of the proposed method

Preventive and corrective coordinated transient stability dispatch against uncertain wind power [10]

- **Results: gen redispatch and load shedding**

- The base case cost is 39173.9 $/Hr;
- For #2, The system is stabilized with both -0.38 and -0.187 risk coordination parameters;
- For -0.38, only PC is adequate and there is no CC. The total is 39190.7 $/Hr, which is only 0.043% higher;
- However, the optimal cost happens when τ is -0.187, which is 39183.9 $/Hr, i.e., only 0.025% increment on the cost can improve the transient stability

Base case dispatch

Generator	G30	G31#	G32*	G33	G34
P_G	239.5	560.9	650.0	624.0	504.1
Generator	G35	G36	G37*	G38	G39*
P_G	644.9	553.0	540	822.4	1000.0
Total cost	39173.9$/Hr				

Robust Generation Dispatch for #2

Generator	G30	G31#	G32*	G33	G34
P_G	235.4	552.3	650.0	639.1	513.8
Generator	G35	G36	G37*	G38	G39*
P_G	651.2	563.6	540	793.2	1000.0
Total cost	39190.7$/Hr				

Generation rescheduling and load shedding for three contingencies

Contingency	τ	Amount of load shedding (MW)	PC cost ($)	CC cost ($)	Total coordination cost ($)
#1	-1	ΔP_{D12}=7.5, ΔP_{D39}=69	39256.3	77.5	39332.8^
#2	-0.38	N/A	39190.7	0	39190.7
	-0.187	ΔP_{D12}=6.0	39177.9	6	39183.9^
#3	-0.907	ΔP_{D12}=7.5, ΔP_{D39}=0.61	39243.9	8.1	39252^

✓ Conclusion:

Corrective control may not be needed to improve the stability since preventive control is enough, but the min cost comes from optimal coordination of PC and CC.

Preventive and corrective coordinated transient stability dispatch against uncertain wind power [10]

- **Results:** **robustness checking**

$$Robustness\ dgree:\ \ \gamma = \frac{M_s}{M} \times 100\%$$

Quantify the robustness of the solutions under uncertainties of wind power variation

Stability margin for base case

Contingency	C1	C2	C3
Stability margin	-71.6	-16.5	-68.3
Robustness	0%	0%	0%

The system is unstable and the robustness is very low for the base case

Single contingency

Transient stability margin for three contingencies

Fault Scenarios	C1	C2	C3
l_1	14.05	13.2	15.53
l_2	1.59	0.36	0.41
l_3	2.15	1.00	1.27
l_4	5.01	3.22	3.96
Robustness	98.1%	93.3%	93%

Multi-contingency

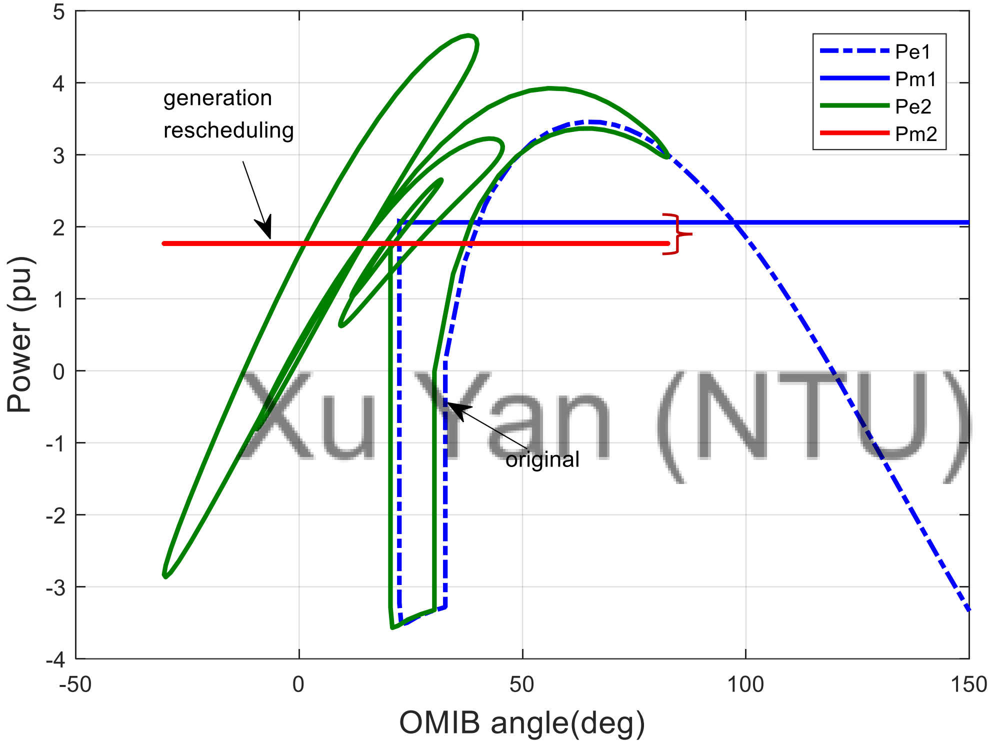
Transient stability margin for multi-cont.

Fault Scenarios	C1	C2	C3
l_1	13.95	41.14	26.6
l_2	1.5	35.51	16.08
l_3	2.07	35.73	16.7
l_4	4.89	37.12	18.92
Robustness	98%	100%	100%

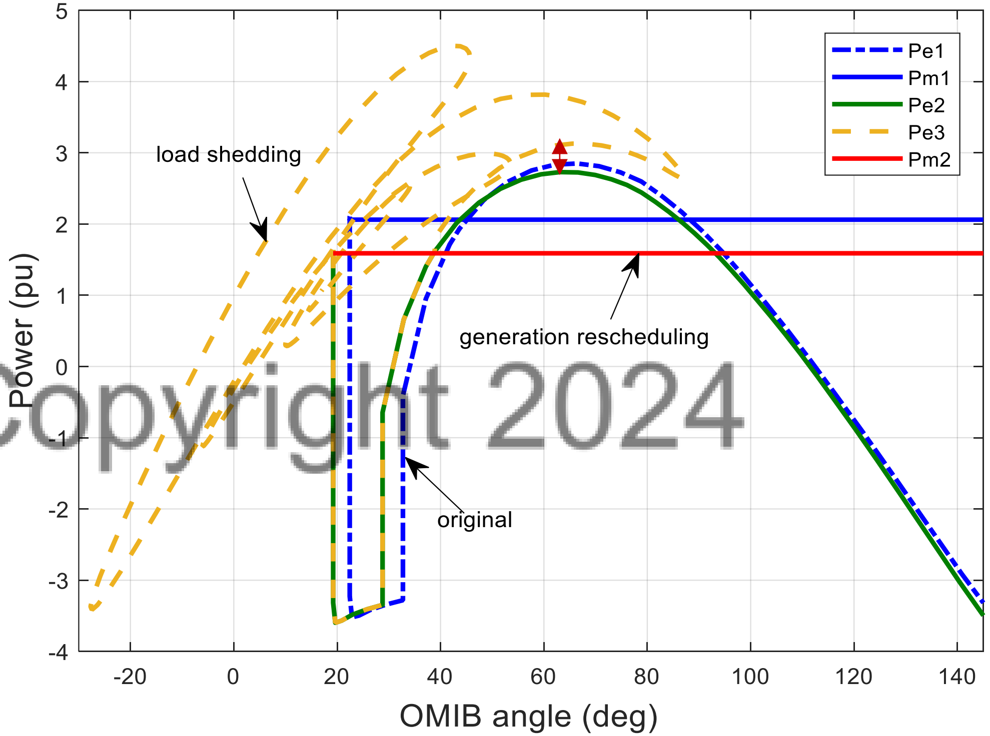
The solution can achieve a high robustness (higher than 90%) against wind uncertainty for both single contingency case and multi-contingency case

Preventive and corrective coordinated transient stability dispatch against uncertain wind power [10]

- **Results: observations from OMIB plane**



Pe-OMIB angle plane for #2 under only PC control

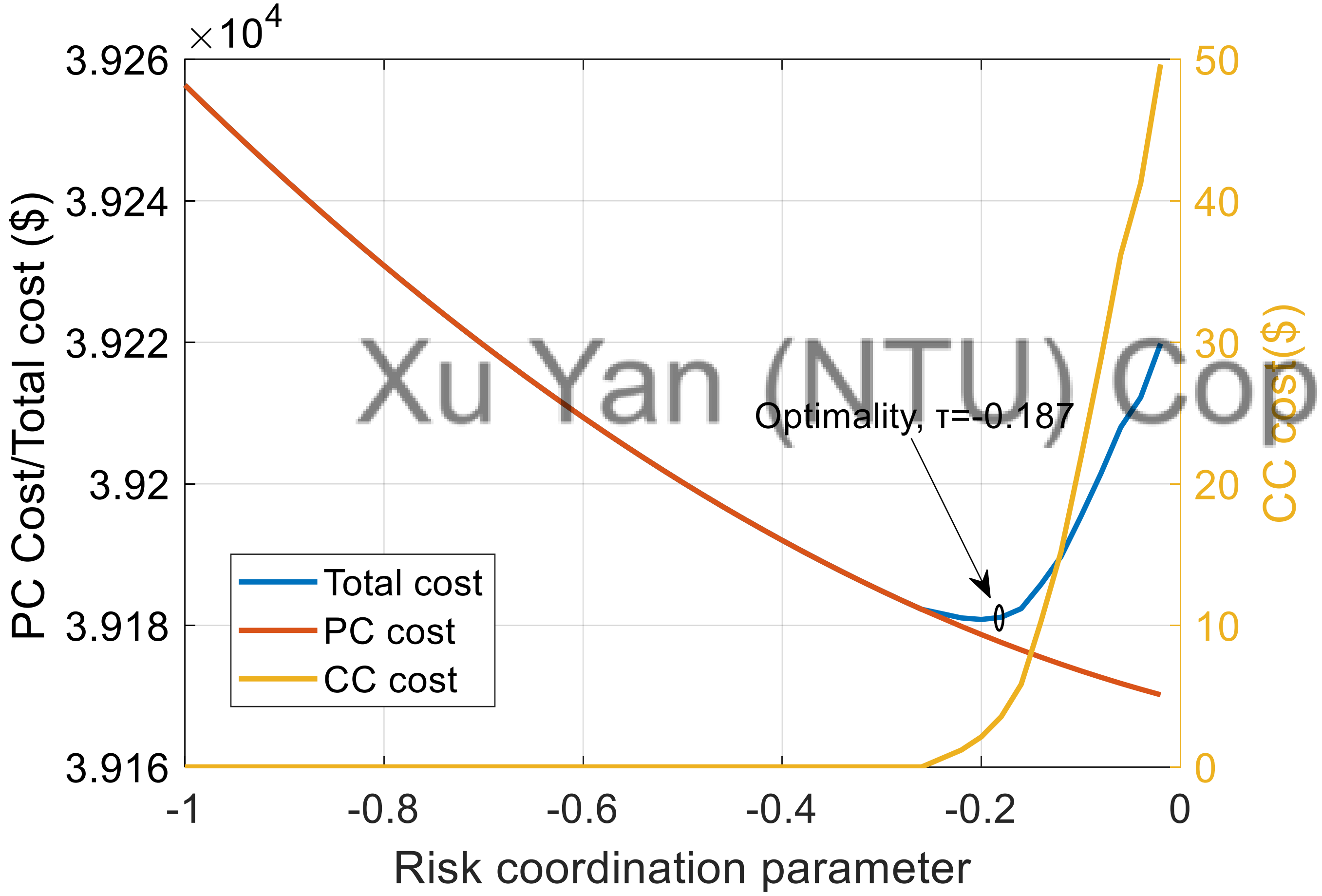


Pe-OMIB angle plane for #3 under both PC and CC control

- Compared red solid and blue solid lines in the left figure, Generation rescheduling is to reduce the mechanical power, which shrinks the accelerating area;
- In the right figure, generation rescheduling alone is not adequate to stabilize the system;
- Compared the green solid and yellow dashed lines in the right figure, load shedding is to increase the electrical power, which enlarges the deaccelerating area;

Preventive and corrective coordinated transient stability dispatch against uncertain wind power [10]

- **Results: solution optimality and speed**



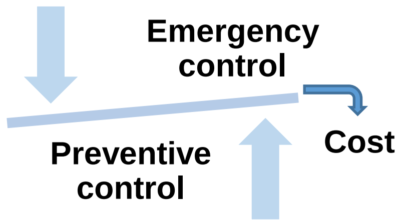
Cost function with respect to the risk coordination parameters for #2

- τ determines the feasible stability region in the PC. When it is large, the stability region becomes large and the solution for PC is close to marginal stability and inexpensive. Meanwhile, the successive CC will be costly.
- PC cost monotonously decreases and CC cost monotonously increases with the increment of τ. The optimal coordination of PC and CC can be gained by the golden section search since the optimized problem is unimodal.
- For a single contingency case, the total computational time is 1080s. For the multi-contingency case, the total computation time is about 1573s. The iterations of the searching process are 30.

Fully Robust Coordination of Generation Dispatch and Load Shedding against Instability with Wind Power [11]

- Motivation: to achieve **fully robustness** (note that all the existing works are partially robust)

Robustness of the solution



In our previous works:

- Coordination is realized
- Cost efficiency is verified

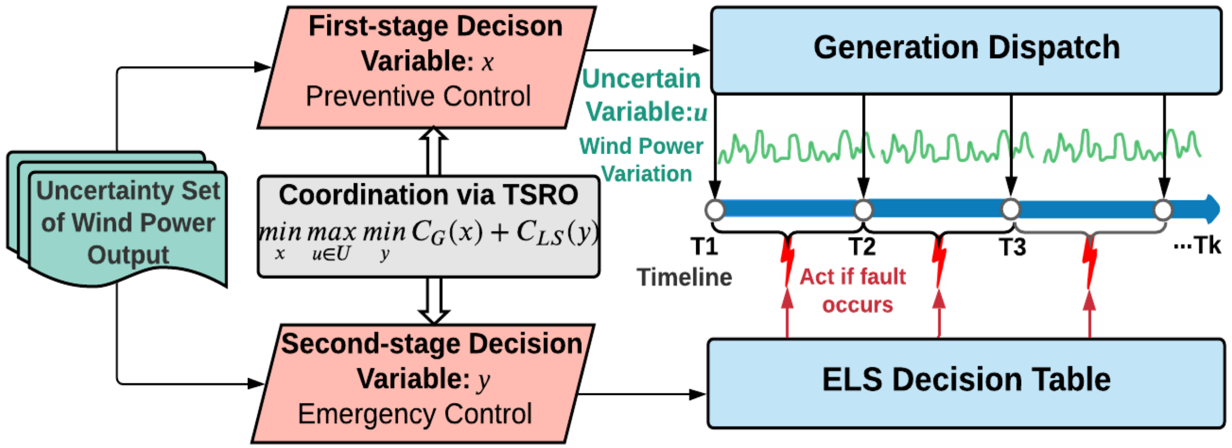
However, the robustness of solutions is not 100% stable against wind power variation, i.e., only partial robustness is achieved.

Fault Scenarios	C1	C2	C3
l_1	14.05	13.2	15.53
l_2	1.59	0.36	0.41
l_3	2.15	1.00	1.27
l_4	5.01	3.22	3.96
Robustness	98.1%	93.3%	93%

Fully robustness

Robust optimization is to find the worst-case scenario of uncertainties and aims to ensure full robustness against uncertainties.

Therefore, to achieve **fully robust stability** of power system against wind power variation, a two stage robust optimization model is proposed:



[11] H. Yuan, Y. Xu, and C. Zhang, "Robustly Coordinated Generation Dispatch and Load Shedding for Power Systems against Transient Instability under Uncertain Wind Power," *IEEE Trans. Power Syst.*, 2022. – the 1st truly robust optimization method for TSCOPF with wind power uncertainty.

Fully Robust Coordination of Generation Dispatch and Load Shedding against Instability with Wind Power [11]

- **Mathematical model: Two Stage Robust Optimization Model (TSRO)**

Compact model

$$\min_{x}\max_{u\in U}\min_{y\in\emptyset_k(x,u)} C_G(x)+C_{LS}(y)$$

$$s.t.\quad g_0(x,y_0,u)=0$$

$$h_0(x,y_0,u)\le 0$$

$$\emptyset_k(x,u)=\left\{\begin{matrix} y: g_k(x,y_k,u)=0, \\ h_k(x,y_k,u)\le 0, \eta_k(x,y_k,u)\ge\sigma \end{matrix}\right\}$$

- Generation dispatch is first stage, modelled in an OPF;
- Emergency load shedding is second stage;
- Stability constraint is a function of generation and load shedding;
- Uncertainty is modelling by the uncertainty set;

Detailed model

$$\min_{P_G}\max_{\tilde{P}_w}\min_{P_L} C_G + C_{LS}$$

$$s.t.\quad C_G=\sum_{i}^{N_G} a_iP_{Gi}^2+b_iP_{Gi}+c_i$$

$$C_{LS}=\sum_{i}^{N_D} p_k C_{LS,O} P_{Li}$$

$$P_{Gi}^t+\tilde{P}_{wi}^t-P_{Di}^t=V_i^t\sum_{j=1}^{N_B}V_j^t\left(G_{ij}\cos\theta_{ij}^t+B_{ij}\sin\theta_{ij}^t\right)$$

$$Q_{Gi}^t+Q_{wi}^t-Q_{Di}^t=V_i^t\sum_{j=1}^{N_B}V_j^t\left(G_{ij}\cos\theta_{ij}^t-B_{ij}\sin\theta_{ij}^t\right)$$

where $t\epsilon[0,T]$

First stage

$$Q_{wi}^t=\tilde{P}_{wi}^t\tan(\cos^{-1}\delta), t\in[0,T]$$

$$Q_{wi}^{\min}{}_{wi}^{t}{}_{wi}^{\max}$$

$$\left\{\begin{matrix} P_{Gi}^{\min}\le P_{Gi}^t\le P_{Gi}^{\max}, & i=1,2,\dots,N_G \\ Q_{Gi}^{\min}\le Q_{Gi}^t\le Q_{Gi}^{\max}, & i=1,2,\dots,N_G \\ V_i^{\min}\le V_i^t\le V_i^{\max}, & i=1,2,\dots,N_B \\ L_i^{\min}\le L_i^t\le L_i^{\max}, & i=1,2,\dots,N_L \end{matrix}\right. \quad t\in[0,T]$$

$$\left\{\begin{matrix} P_{Di}^t=(P_{Di}^0-P_{Li}) \\ Q_{Di}^t=(Q_{Di}^0-Q_{Li}) \end{matrix}\right. t\in[0,T]$$

$$P_{Di}^0\ge P_{Li}\ge 0$$

$$Q_{Li}=P_{Li}\tan(\cos^{-1}\vartheta)$$

Second stage

$$o_k(P_G,P_L)\ge\sigma$$

Stability constraints

Uncertainty Set

$$U_w=\left\{\tilde{P}_{wi}\in\Re^{i_w}:\tilde{P}_{wi}^{\min}\le\tilde{P}_{wi}\le\tilde{P}_{wi}^{\max},\mu_{w.l}\le\frac{\sum_{i\in N_W}\tilde{P}_{wi}}{\sum_{i\in N_W}\tilde{P}_{wi}^{pr}}\le\mu_{w,u}\quad\forall i\right\}$$

$\mu_{w,l}$ and $\mu_{w,u}$ limit the overall uncertainty degree of the total wind power outputs, i,e., overall possible realization of wind power output at different locations is limited

Fully Robust Coordination of Generation Dispatch and Load Shedding against Instability with Wind Power [11]

- **Stability constraint construction**

Recall of trajectory sensitivity

$$\phi(x_0,t,\beta+\Delta\beta)=\phi(x_0,t,\beta)+\frac{\partial\phi(x_0,t,\beta)}{\partial\beta}\Delta\beta+\varepsilon^{\phi}$$

$$\Delta x(t)=\phi(x_0,t,\beta+\Delta\beta)-\phi(x_0,t,\beta)$$

$$\approx\frac{\partial\phi(x_0,t,\beta)}{\partial\beta}\Delta\beta\equiv\Phi(x_0,t,\beta)\Delta\beta$$

$$\Phi(x_0,t,\beta)=\frac{\phi(x_0,t,\beta+\Delta\beta)-\phi(x_0,t,\beta)}{\Delta\beta}$$

$$\Delta\beta=\frac{\Delta\eta}{\Phi(\eta,\beta)}$$

Transient stability constraint construction

$$\eta_k(PC,EC,\Delta P_w)=\eta_k^{PC}+\Delta\eta_k^{\Delta P_w}+\Delta\eta_k^{EC}\geq\sigma$$

$$\Delta\eta_k^{\Delta P_w}=f_1(\Delta\tilde{P}_w)$$

$$\Delta\eta_k^{EC}=f_2(P_{Li})$$

$$\sum_i^{N_G}\Delta P_{Gi}=-\sum_i^{N_W}\Delta\tilde{P}_{wi}=-\Delta\tilde{P}_w$$

$$\Delta P_{Gi}=-\kappa_i\Delta\tilde{P}_w\, i=1,2,3....N_G$$

$$\Delta\eta_k^{\Delta P_w}=f_1(\Delta\tilde{P}_w)=-\sum_i^{N_G}\Phi_i(\eta,P_{Gi})\kappa_i\Delta\tilde{P}_w$$

$$\Delta\eta_k^{EC}=f_2(P_{Li})=\sum_i^{N_D}\Phi_i(\eta,P_{Li})P_{Li}$$

$$o_k(P_G,P_L)\geq\sigma$$

$$\eta_k^{PC}-\sum_i^{N_G}\Phi_i(\eta,P_{Gi})\kappa_i\Delta\tilde{P}_{wi}+\sum_i^{N_D}\Phi_i(\eta,P_{Li})P_{Li}\geq\sigma$$

Final linear form of stability constraint

- Function 1 and function 2 is related to the wind power and load shedding, respectively;
- Wind power is balanced by synchronous generation with an AGC factor
- By trajectory sensitivity, stability constraints is constructed by linear form;

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Fully Robust Coordination of Generation Dispatch and Load Shedding against Instability with Wind Power [11]

- **Solution approach**

Column and Constraint Generation (CC&G)

TSRO matrix model

$$\min_{x}\max_{u\in U_w}\min_{y} ax^2+bx+c+d^Ty$$
$$s.t.\ Ax+Bu=b$$
$$Fx\le f$$
$$Gy\le g$$
$$U(x)y+V(x)u\le e$$
$$y\ge 0$$
$$u\in U_w$$

Master problem

$$\min_{x} ax^2+bx+c+\theta$$
$$s.t.\ Ax+Bu_l^*=b$$
$$Fx\le f$$
$$\theta\ge d^Ty_l$$
$$Gy_l\le g$$
$$U(x)y_l+V(x)u_l^*\le e\forall u_l^*\in S_l$$
$$y_l\ge 0$$

Sub-problem

$$S(u,x^*)=\max_{u}\min_{y} d^Ty$$
$$s.t.\ Gy\le g$$
$$U(x^*)y+V(x^*)u\le e$$
$$y\ge 0$$
$$u\in U_w$$

Strong Duality: $min \longrightarrow max$

$$R(x)=\max_{u\in U,\lambda,\rho} -\lambda g+\rho^T(Vu-e)$$
$$d^T+G^T\lambda+U^T\rho\ge 0,\lambda\ge 0,\rho\ge 0$$

- The *min-max-min* problem is decomposed to a master problem and a sub-problem.
- $R(x)$ is a NP-hard problem due to the bilinear term "ρ^TVu".
- OA (outer approximation) algorithm is used to solve the bilinear problem.

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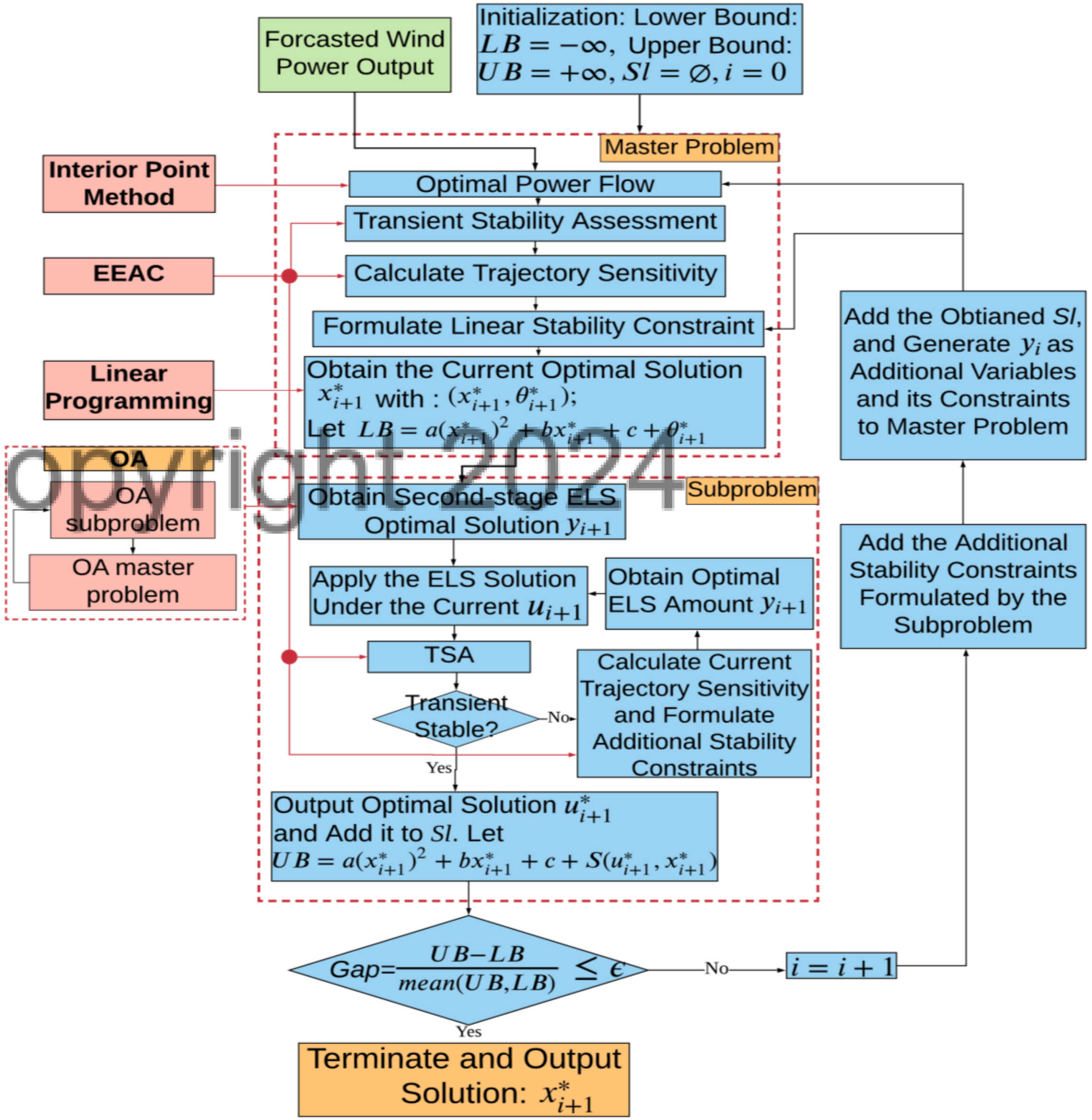
- **Computation process**

  - The modified C&CG algorithm has two iteration loops:

1) Outer loop: conventional C&CG iteration between the master problem and the sub-problem
2) Inner loop: stability checking iteration in the subproblem

Contingency set for New England 39-bus System

Contingency ID	Fault bus	Fault clearance	Tripped line
C1	22	0.25 s	22-23
C2	29	0.10 s	26-29
C3	26	0.15 s	26-28



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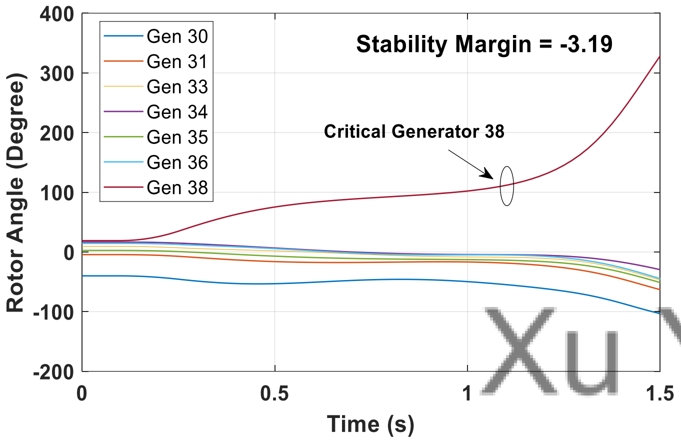
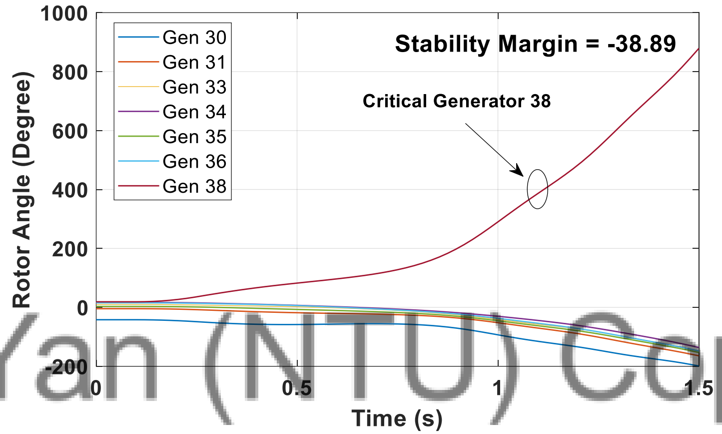
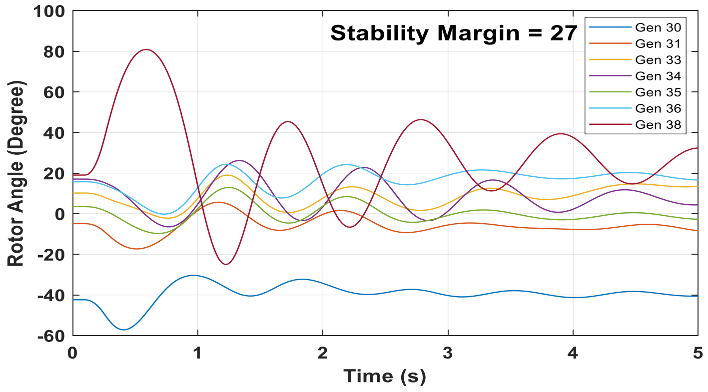
- **Results: operation solution**



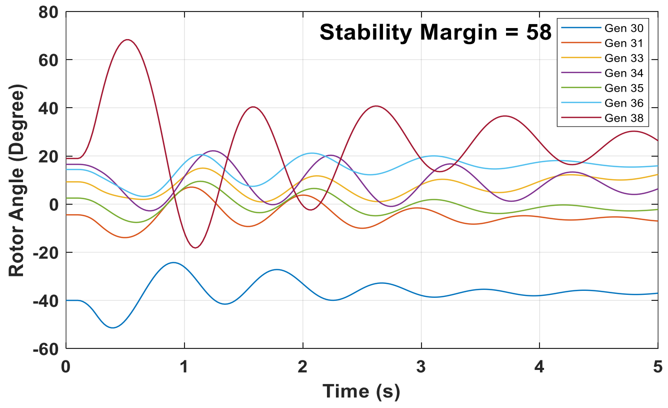
Rotor-angle curve under initial case



Rotor angle curve under worst-case without ELS



Rotor angle curve under worst-case with ELS



Rotor angle curve under a non-worst case with ELS

Four figures are the rotor angle curves for C3:

- The system is unstable under base case (-3.19);
- The system becomes more unstable under worst-case without emergency control (-38.89);
- The system becomes stable with emergency control (27.0) under the worst case;
- The system is more stable under a non-worst case with emergency control (58.0);
- The emergency control is effective and the worst-case is found.

Load shedding amount with 0.9-1.1 uncertainty budget

Contingency ID	Amount of load shedding (MW)		
C1	P_{L15}=154.8		
C2	P_{L25}=141.4	P_{L26}=139	P_{L27}=281
C3	P_{L25}=72.6	P_{L26}=139	P_{L27}=281

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- **Results: robustness check**

Robustness dgree: $\gamma = \frac{M_s}{M} \times 100\%$

Quantify the robustness of the solutions under uncertainties of wind power variation

Stability margin for base case

Contingency	C1	C2	C3
Stability margin	27.27	-21.49	-3.19
Robustness	91.4%	17.4%	43.3%

The system is unstable and the robustness is low for base case

Solutions with largest uncertainty budget pair can achieve 100% stability robustness against wind power uncertainty

Robustness check with different uncertainty budget pair for C1

Method	Proposed method			Deterministic
Uncertainty budget pair	1	2	3	N. A
$\mu_{w,l}$	0.95	0.9	0.85	
$\mu_{w,u}$	1.05	1.1	1.15	
Total cost under worst case ($/Hr)	40969.9	44323	51247	39173.9
Stability robustness degree check (%)	MCS Group 1: $\pm5\%\tilde{P}_w^{pr}$			
	100%	100%	100%	100%
	MCS Group 2: $\pm10\%\tilde{P}_w^{pr}$			
	99.4%	100%	100%	99.4%
	MCS Group 3: $\pm15\%\tilde{P}_w^{pr}$			
	91.4%	97.3%	100%	91.4%

Robustness check with different uncertainty budget pair for C2

Method	Proposed method			Deterministic
Uncertainty budget pair	1	2	3	N. A
Total cost under worst case ($/Hr)	44317	48388	53598	40708.91
Stability robustness degree check (%)	MCS Group 1: $\pm5\%\tilde{P}_w^{pr}$			
	100%	100%	100%	54.5%
	MCS Group 2: $\pm10\%\tilde{P}_w^{pr}$			
	97.9%	100%	100%	49.9%
	MCS Group 3: $\pm15\%\tilde{P}_w^{pr}$			
	86.7%	100%	100%	50.1%

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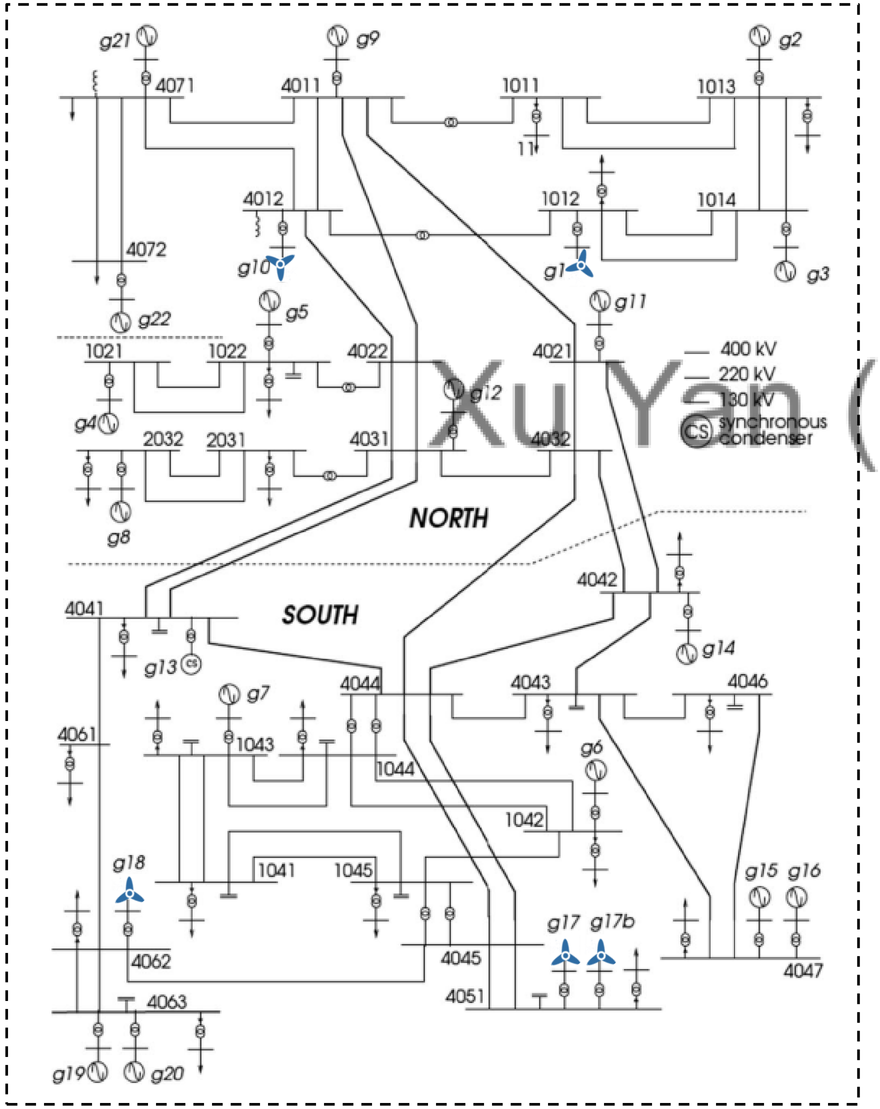
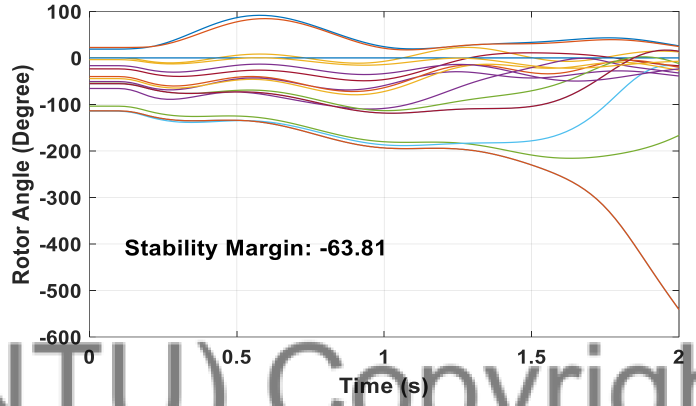
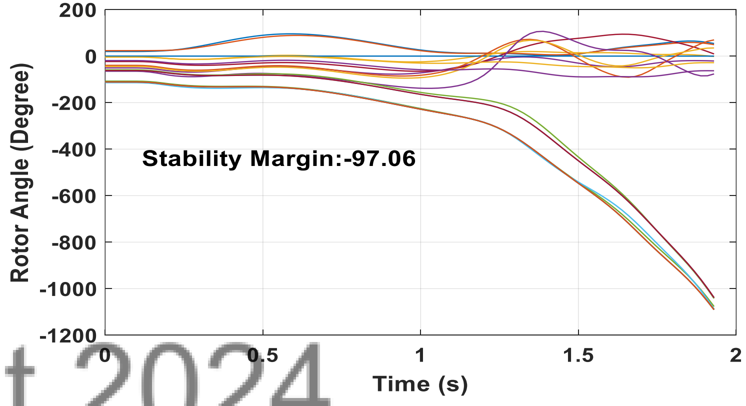
- **Results: Nordic 32 test system**

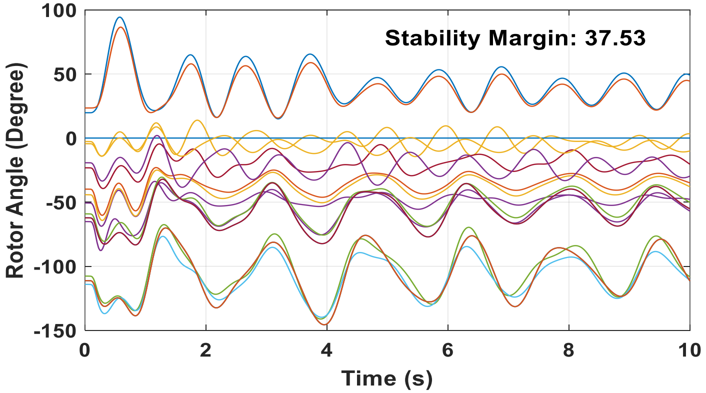




Rotor-angle curve under initial case for Nordic system



Rotor-angle curve under worst-case without ELS for Nordic System



Rotor angle curve under worst-case with ELS for Nordic System

The system is unstable under base case (-63.81), and become even more unstable under worst-case (-97.06), and change to be stable with emergency control (37.53)

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• Results

Robustness check with different uncertainty budget pair of Nordic system

Method	Proposed method		Deterministic
Uncertainty budget pair	1	2	N. A
$\mu_{w,l}$	0.85	0.95	
$\mu_{w,u}$	1.15	1.05	
Total cost under worst case ($/Hr)	246845.54	240553.59	235780.33
Stability robustness degree check (%)	MCS Group 1: $\pm 5\% \tilde{P}_w^{pr}$		
	100%	100%	62.2%
	MCS Group 2: $\pm 15\% \tilde{P}_w^{pr}$		
	100%	99.9%	55.8%

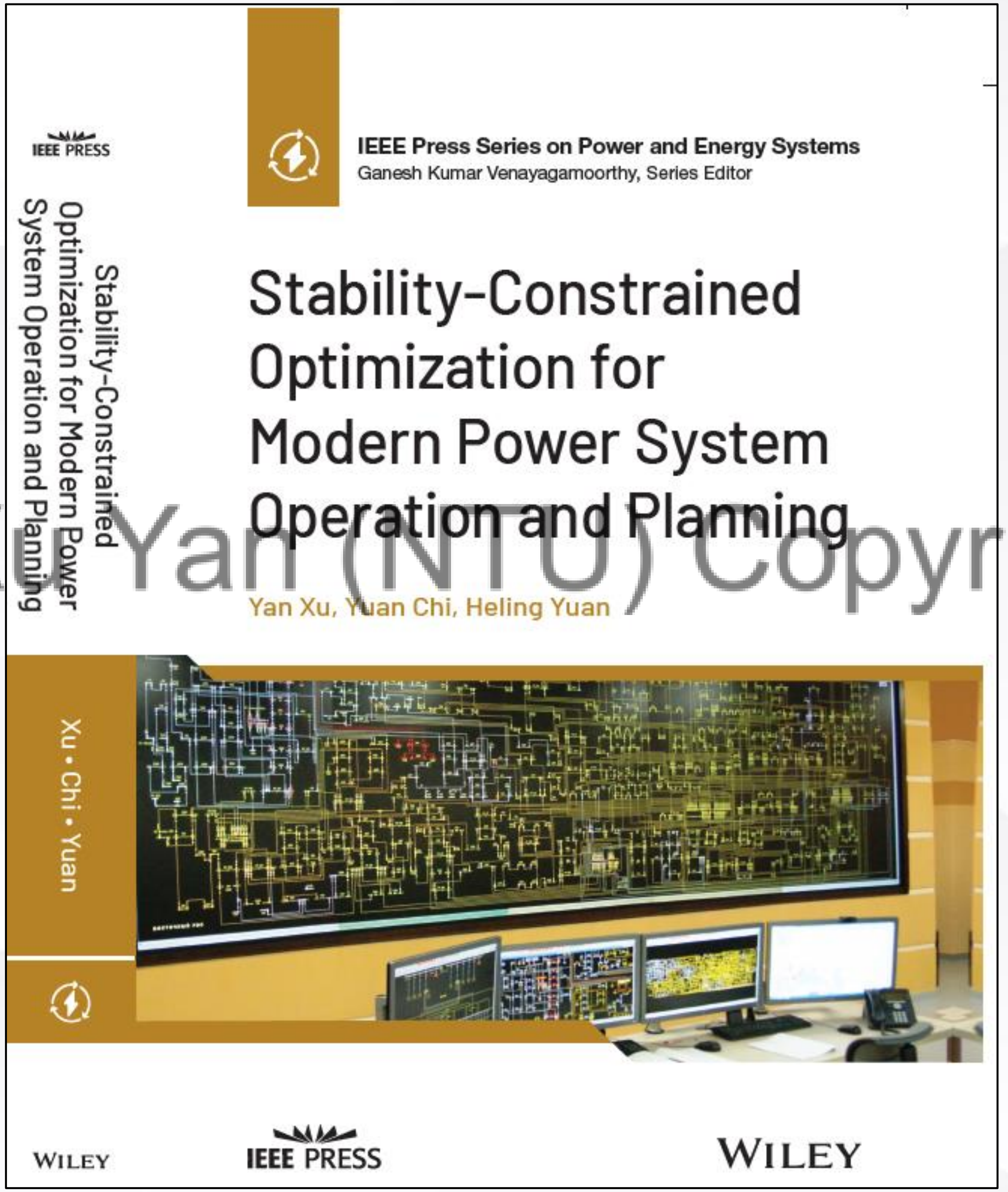
The robustness of the system is still 100% against wind uncertainty with the largest uncertainty budget pair.

Time consumption for different calculation tasks

Calculations	Cases	TDS	Stability margin	Trajectory sensitivity		Solver time of master and sub-problem	Iteration No.	Total programming time
				Gen	Load			
New-England	C1	0.3s	negligible	25s	39s	2s	6	278s
	C2						4	176s
	C3						2	101s
Nordic	\	0.7s	negligible	53s	47s	1.3s	2	170s

- Computational burden comes from the trajectory sensitivity calculation;
- Meanwhile, the number of iterations also affect the computation time.

Our book on Stability-Constrained Optimization for Power System



Y. Xu, Y. Chi, and H. Yuan, "***Stability-Constrained Optimization for Modern Power System Operation and Planning***," IEEE-Wiley Press, 2023.

This book is a systematic presentation of our original research works on stability-constrained power system optimization, including:

1) transient stability-constrained dispatch and operational control, and
2) dynamic VAR resources placement for power system voltage stability enhancement.

https://www.wiley.com/en-us/Stability+Constrained+Optimization+for+Modern+Power+System+Operation+and+Planning-p-9781119848868

https://www.amazon.com/Stability-Constrained-Optimization-Operation-Planning-Systems/dp/1119848865

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ROLLS ROYCE

Stability-Constrained Load Restoration Considering Multi-phase Cold Load Pickup Effects

[12] D. Xie, Y. Xu, S. Nadarajan, V. Viswanathan, and A.K. Gupta, "Dynamic Frequency-Constrained Load Restoration Considering Multi-Phase Cold Load Pickup Behaviours," *IEEE Trans. Power Syst.*, 2023.

[13] D. Xie, Y. Xu, S. Nadarajan, V. Viswanathan, and A.K. Gupta, "A Transparent Data-Driven Method for Stability-Constrained Load Restoration Considering Multi-Phase Load Dynamics," *IEEE Trans. Power Syst.*, 2023.

(To be introduced in future presentations)

Thank You!